Special Matrices

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Transpose

- For any matrix $A_{m,n}$ its **transpose** is called A^T
- A^T is a matrix with n rows and m columns
- Element $(A)_{ij}$ in A corresponds to element $(A^T)_{ji}$ in A^T
- In R:

```
(A <- matrix(c(1:6), nrow = 3, ncol = 2, byrow = TRUE))
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    3    4
## [3,]    5    6
```

```
t(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    5
## [2,]    2    4    6
```

Symmetric Matrices

- Matrices which are identical to its transpose are called **symmetric**
- Hence for a symmetric matrix S

$$S = S^T$$

```
S
```

```
##      [,1] [,2]
## [1,]   35   44
## [2,]   44   56
```

```
t(S)
```

```
##      [,1] [,2]
## [1,]   35   44
## [2,]   44   56
```

Identity Matrix

- Name: I_n
- Dimensions: $n \times n$
- Example $n = 3$

$$I_3 = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

- In R:

```r
diag(1, nrow = 3, ncol = 3)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

Importance of I_n

- Neutral element of matrix multiplication
- For any matrix $A_{m,n}$:

$$A \cdot I_n = I_m \cdot A = A$$

Diagonal Matrices

- Matrices with non-zero elements only on diagonal
- Diagonal matrices are symmetric
- In R:

```r
D <- diag(c(5,8,-3))
D
```

```
##      [,1] [,2] [,3]
## [1,]    5    0    0
## [2,]    0    8    0
## [3,]    0    0   -3
```

Inverse Matrix

- The matrix B is called the **inverse** of matrix A, if their product $A \cdot B$ is the identity matrix I

$$A \cdot B = I$$

* If it exists, the inverse matrix of matrix A is called A^{-1} * In R the inverse of matrix S is computed as:

```
solve(S)
```

```
##            [,1]      [,2]
## [1,]  2.333333 -1.833333
## [2,] -1.833333  1.458333
```