$$
\begin{aligned}
var(m_i) &= var(a_i) - \frac{1}{4}var(a_s) - \frac{1}{4}var(a_d) - \frac{1}{2}cov(a_s, a_d) \\
&= (1 + F_i)\sigma_a^2 - \frac{1}{4}(1 + F_s)\sigma_a^2 - \frac{1}{4}(1 + F_d)\sigma_a^2 - \frac{1}{2} * 2 * F_i * \sigma_a^2 \\
&= \left(\frac{1}{2} - \frac{1}{4}(F_s + F_d)\right)\sigma_a^2 \qquad (6.9)
\end{aligned}
$$

In case where only parent s of animal i is known the terms in (6.10) and (6.9) change to

$$a_i = \frac{1}{2}a_s + \frac{1}{2}a_d + m_i \qquad (6.10)$$

$$
\begin{aligned}
a_i &= \frac{1}{2}a_s + m_i \\
var(m_i) &= \left(1 - \frac{1}{4}(1 + F_s)\right)\sigma_a^2 \\
&= \left(\frac{3}{4} - \frac{1}{4}F_s\right)\sigma_a^2 \qquad (6.11)
\end{aligned}
$$

When both parents are unknown, we get

$$
\begin{aligned}
a_i &= m_i \\
var(m_i) &= \sigma_a^2 \qquad (6.12)
\end{aligned}
$$

6.6.3 Decomposition of A

The true breeding values a_s of sire s and a_d of dam d can be decomposed in a similar way as shown for the true breeding value a_i in (6.10).

$$
\begin{aligned}
a_s &= \frac{1}{2}a_{ss} + \frac{1}{2}a_{ds} + m_s \\
a_d &= \frac{1}{2}a_{sd} + \frac{1}{2}a_{dd} + m_d \qquad (6.13)
\end{aligned}
$$

where

ss	sire of s
ds	dam of s
sd	sire of d
dd	dam of d

Using (6.13) together with (6.10) leads to the following expression for a_i

$$
\begin{aligned}
a_i &= \frac{1}{2}a_s + \frac{1}{2}a_d + m_i \\
&= \frac{1}{2}(\frac{1}{2}a_{ss} + \frac{1}{2}a_{ds} + m_s) + \frac{1}{2}(\frac{1}{2}a_{sd} + \frac{1}{2}a_{dd} + m_d) + m_i \\
&= \frac{1}{4}a_{ss} + \frac{1}{4}a_{ds} + \frac{1}{4}a_{sd} + \frac{1}{4}a_{dd} + \frac{1}{2}m_s + \frac{1}{2}m_d + m_i
\end{aligned}
$$