

Multiple Traits

Peter von Rohr

29. November 2019

So Far ...

- ▶ Prediction of Breeding Values for **one trait**

→ **univariate** analyses

- ▶ In Livestock Breeding, populations are improved with respect to several traits

→ **multi-trait** or multiple trait

- ▶ Different selection strategies and different approaches of how data is analysed are possible

Multiple Trait Selection

- ▶ Selection index theory provides a tool for optimal integration of different sources of information
- ▶ But still other strategies are applied
 - ▶ Tandem selection
 - ▶ Selection based on independent thresholds

Tandem Selection

- ▶ Improve one trait at the time until they all reach a certain threshold
- ▶ Problem: For traits which are not improved
 - ▶ only correlated selection responses
 - ▶ can be negative
- ▶ Populations with long generation intervals, response per year is very small

Independent Selection Thresholds

- ▶ Applied before selection index
- ▶ Define selection thresholds in each of the traits
- ▶ Select animals as parents which are above thresholds for all traits

Pros and Cons

- ▶ Selection response in all traits
- ▶ Thresholds often set to only positive predicted breeding values in all traits

→ exclusion of very many animals and reduction in genetic variability

- ▶ Genetic relationships between traits ignored

→ genetic gain will not be as expected

3. Differences in the economic relevance ignored.

→ threshold in all traits above positive predicted breeding values emphasizes traits with high heritability

Aggregate Genotype

- ▶ Define the set of important traits for which population should be improved
- ▶ Determine economic values w for these traits
- ▶ Aggregate genotype H follows as

$$H = w^T u$$

Selection Index

- ▶ Use index I to estimate H where I is a linear combination of information sources

$$I = b^T \hat{u}$$

- ▶ Index weights b are determined using selection index theory as

$$b = P^{-1} G w$$

- ▶ Information sources are predicted breeding values
- ▶ If traits in u and \hat{u} are the same and \hat{u} were estimated using BLUP, then $b = w$

Implementations

- ▶ First possible implementation
 - ▶ Do univariate predictions of breeding values using BLUP animal model
 - ▶ Combine \hat{u} with appropriate b -values
- ▶ Improvement
 - ▶ get \hat{u} from multivariate analysis

Multivariate Analysis

- ▶ Given two traits with univariate models

$$y_1 = X_1\beta_1 + Z_1u_1 + e_1$$

$$y_2 = X_2\beta_2 + Z_2u_2 + e_2$$

- ▶ Combine both univariate models by stacking one on top of the other, resulting in

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Multivariate Model

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

can be written as

$$y = X\beta + Zu + e$$

$$\text{with } y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}, Z = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$$

Multivariate Variance-Covariance Matrices

$$G_0 = \begin{bmatrix} \sigma_{g_1}^2 & \sigma_{g^1, g^2} \\ \sigma_{g^1, g^2} & \sigma_{g_2}^2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$\text{var}(u) = \text{var} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} g_{11}A & g_{12}A \\ g_{21}A & g_{22}A \end{bmatrix} = G_0 \otimes A = G$$

$$R_0 = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

$$R = \text{var}(e) = \text{var} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} r_{11}I_n & r_{12}I_n \\ r_{21}I_n & r_{22}I_n \end{bmatrix} = R_0 \otimes I_n$$

Solutions

- ▶ Mixed Model Equations

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

Advantages

- ▶ some traits have lower heritability than others
- ▶ environmental correlations exist between traits measured on the same animal
- ▶ some traits are available only a subset of all animals
- ▶ some traits were used for a first round of selection
- ▶ accuracies are higher in multivariate analyses