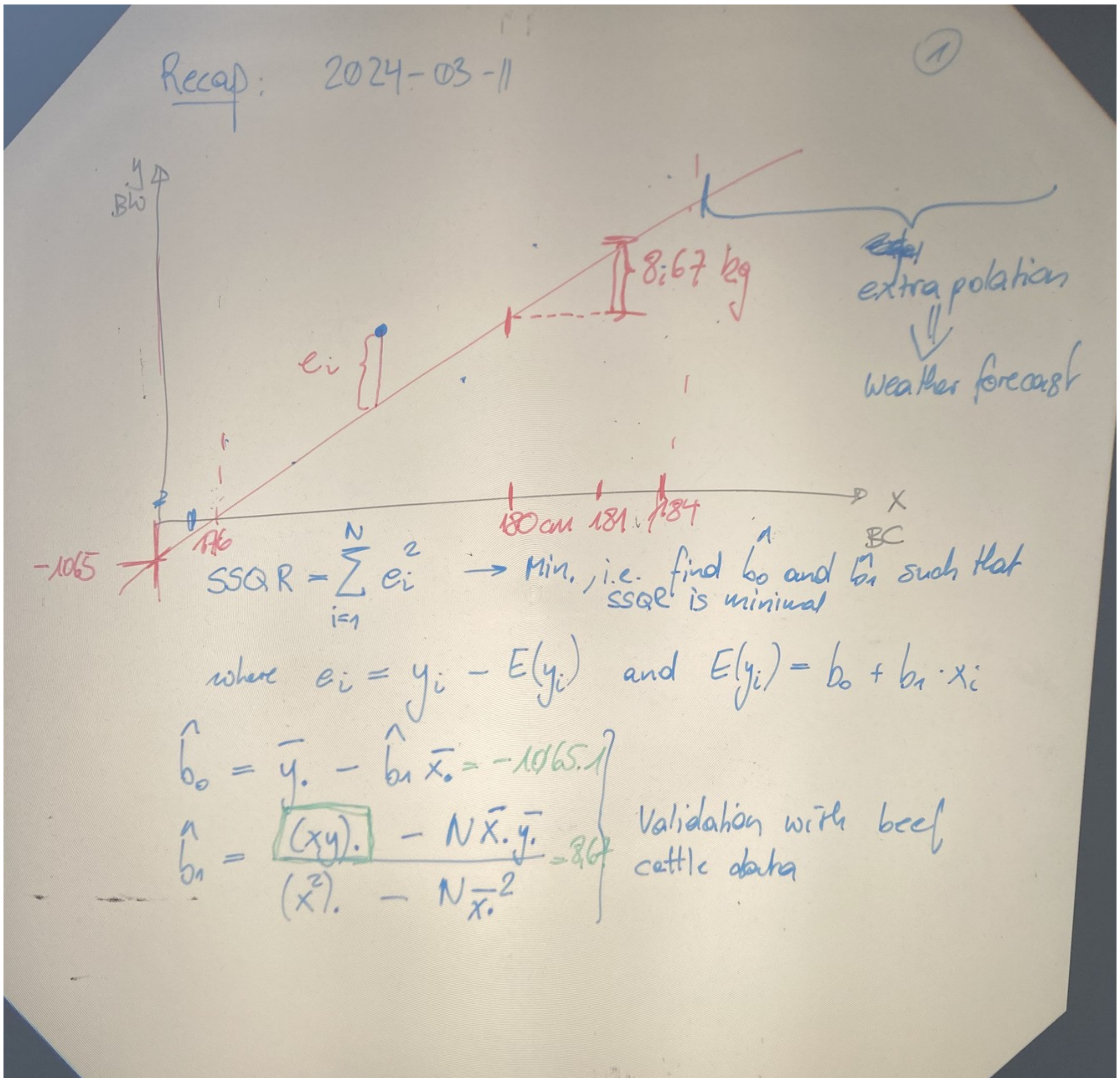
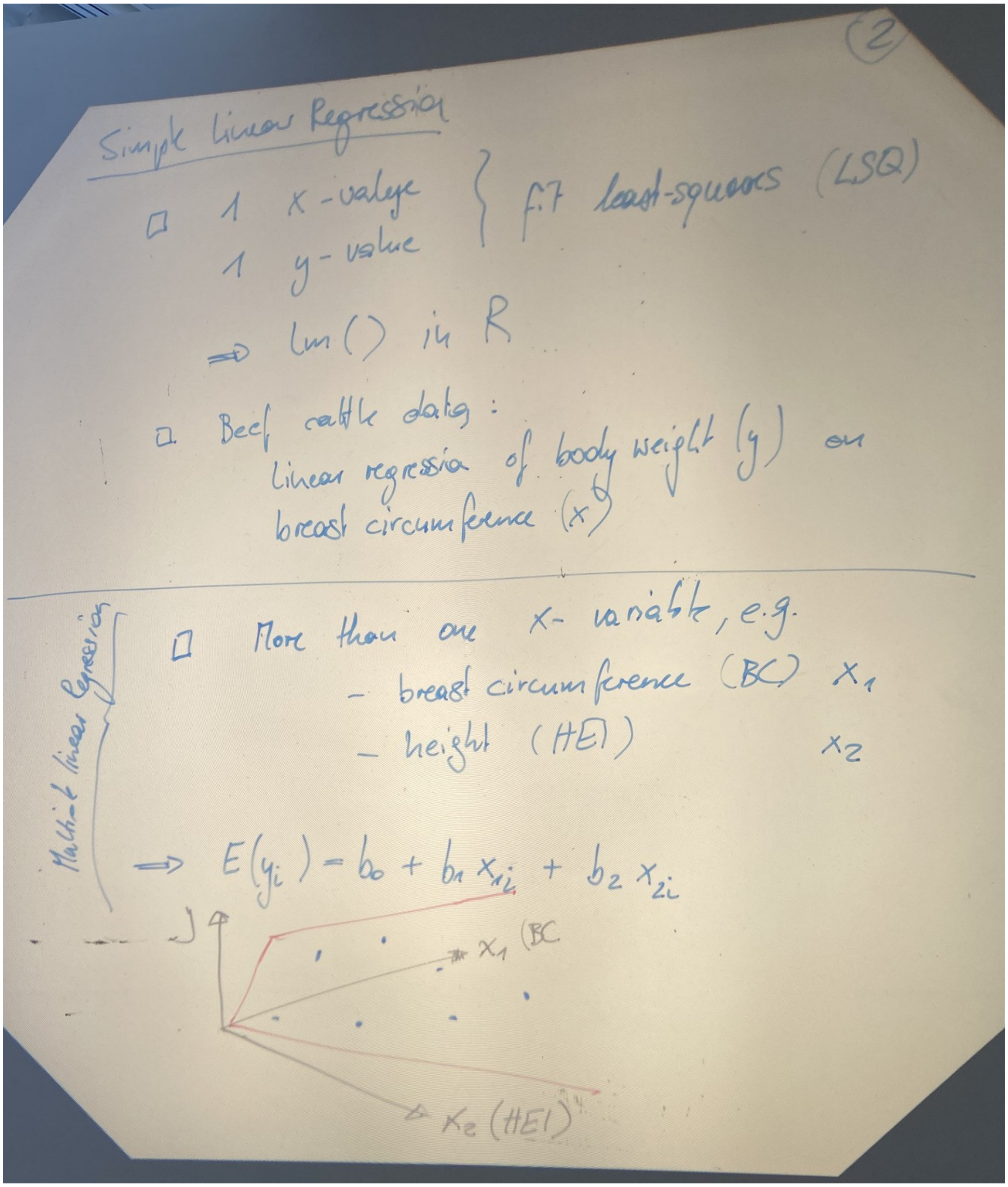
OHP Picture 1



OHP Picture 2



OHP Picture 3

Multiple Linear Regression

- $SSQR = \sum_{i=1}^{N} e_i^2 \quad ; \quad e_i = y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}$

- Goal: Find estimates $\hat{b}_0, \hat{b}_1, \hat{b}_2$ such that SSQR is minimal $\Rightarrow$ Least Squares

$$\Rightarrow \underbrace{\frac{\partial SSQR}{\partial b_0}}_{=0} \; ; \; \underbrace{\frac{\partial SSQR}{\partial b_1}}_{=0} \; ; \; \underbrace{\frac{\partial SSQR}{\partial b_2}}_{=0}$$

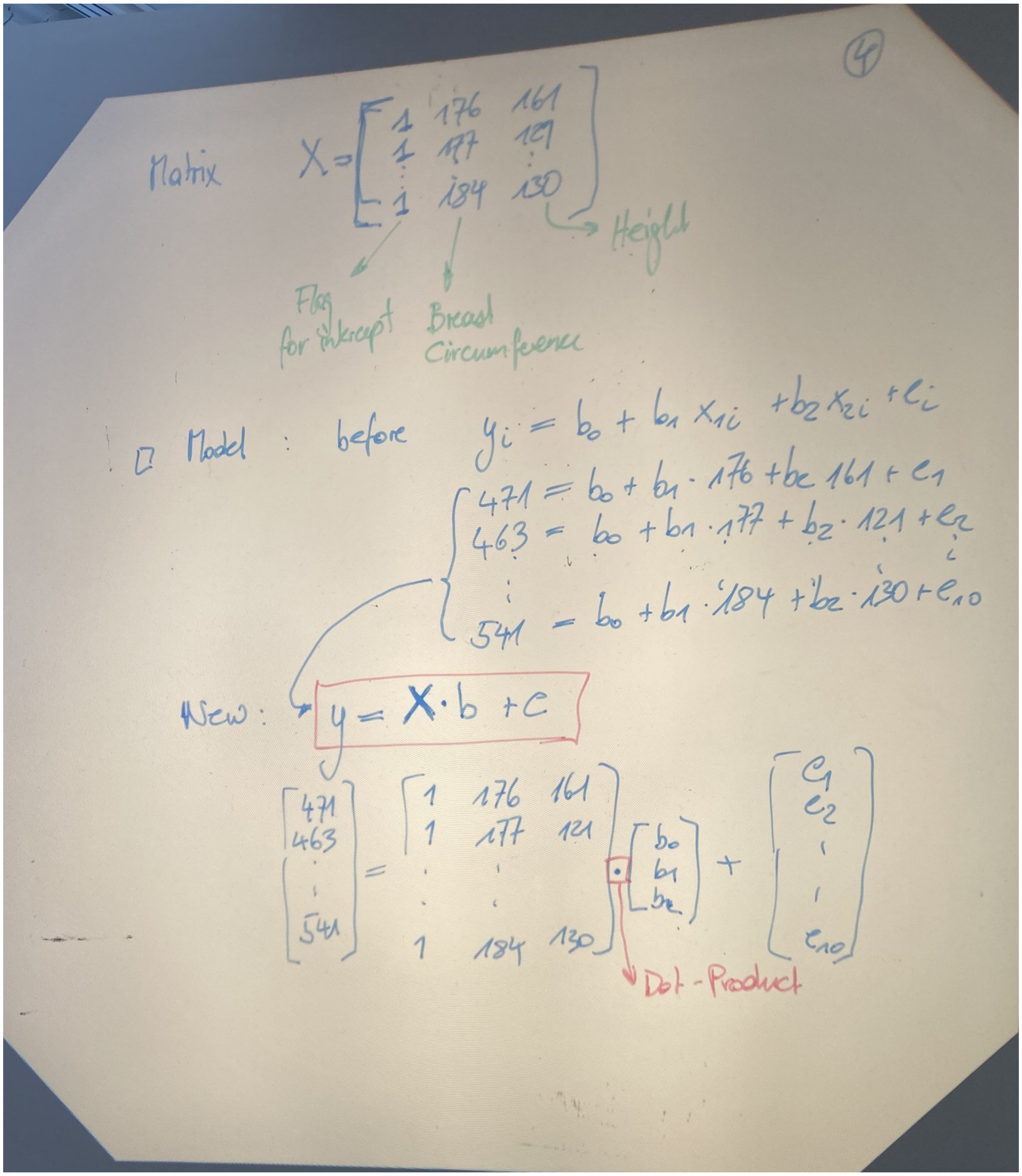
- Matrix-Vector Notation:

Define three vectors y, e and b

$$\left.\text{vector } y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} 471 \\ 463 \\ \vdots \\ 541 \end{bmatrix}\right\} \text{observations of response variable}$$

$$\text{vector } e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} \; ; \; \text{vector } b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \begin{array}{l} \to \text{intercept} \\ \to \text{slope in } x_1 \\ \to \text{slope in } x_2 \end{array}$$

OHP Picture 4



OHP Picture 5

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Least-Squares:

- Before $e_i = y_i - E[y_i]$

$$= y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}$$

$$\vdots$$

- New vector $e = y - X \cdot b$ from $y = Xb + e$ subtract Xb on both sides

- $SSQR = \sum_{i=1}^{N} e_i^2 = e_1^2 + e_2^2 + \dots + e_N^2$

New $SSQR = e^T \cdot e$

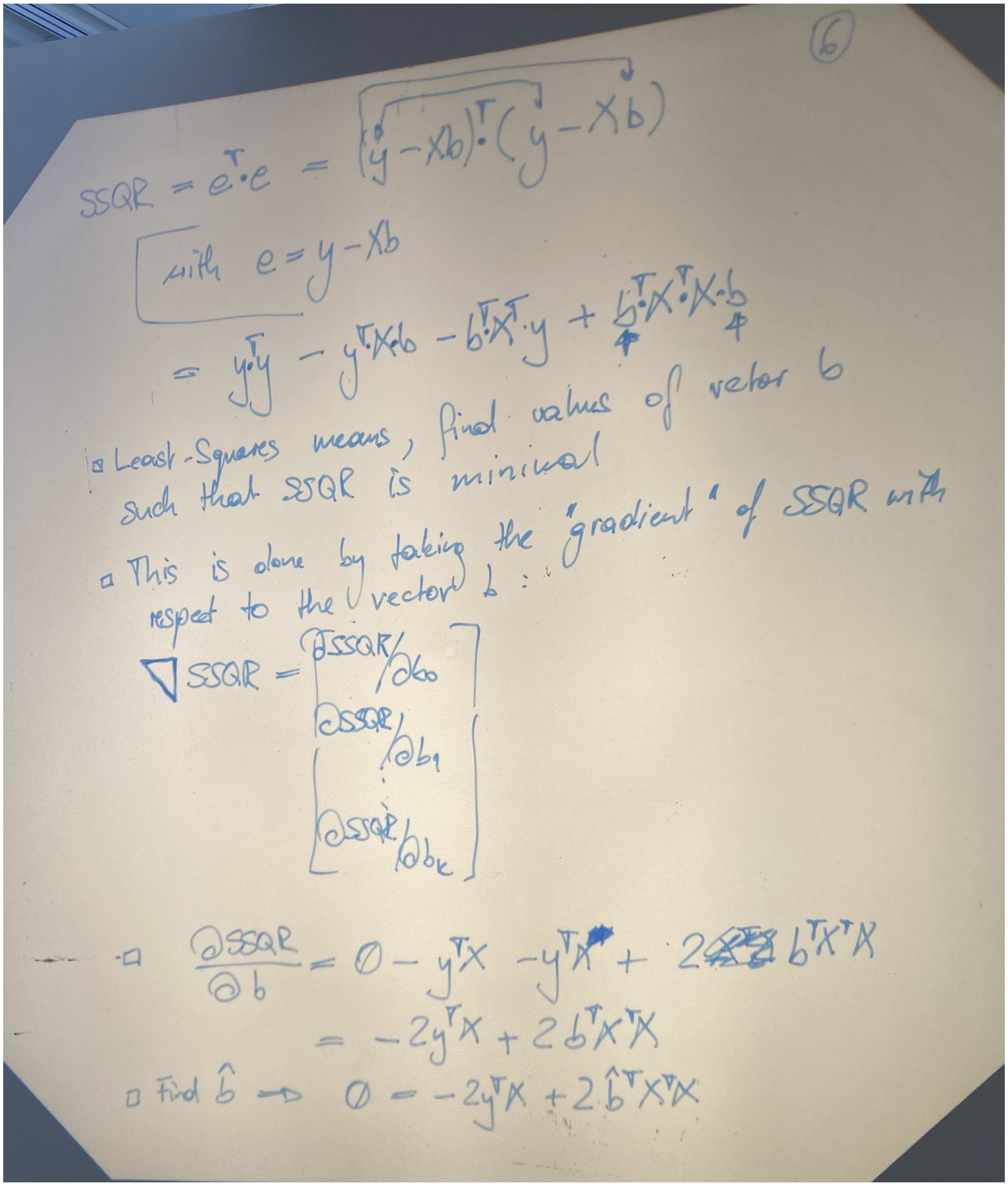
For a colum vector $e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}$ its transpose e^T is defined as the row vector $e^T = \begin{bmatrix} e_1 & e_2 & e_3 & \cdots & e_N \end{bmatrix}$

By definition of dot-product:

$$\boxed{e^T \cdot e} = \begin{bmatrix} e_1 & e_2 & \cdots & e_N \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = e_1^2 + e_2^2 + \dots + e_N^2$$

cross product of e, in R: crossprod()

OHP Picture 6



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$$SSQR = e^T \cdot e = (y - Xb)^T \cdot (y - Xb)$$

$$\boxed{\text{with } e = y - Xb}$$

$$= y^T \cdot y - y^T \cdot X \cdot b - b^T \cdot X^T \cdot y + b^T \cdot X^T \cdot X \cdot b$$

- Least-Squares means, find values of vector b such that SSQR is minimal
- This is done by taking the "gradient" of SSQR with respect to the vector b:

$$\nabla SSQR = \begin{bmatrix} \partial SSQR / \partial b_0 \\ \partial SSQR / \partial b_1 \\ \vdots \\ \partial SSQR / \partial b_k \end{bmatrix}$$

- $$\frac{\partial SSQR}{\partial b} = 0 - y^T X - y^T X + 2 b^T X^T X$$
  $$= -2 y^T X + 2 b^T X^T X$$
- Find $\hat{b} \rightarrow 0 = -2 y^T X + 2 \hat{b}^T X^T X$

OHP Picture 7