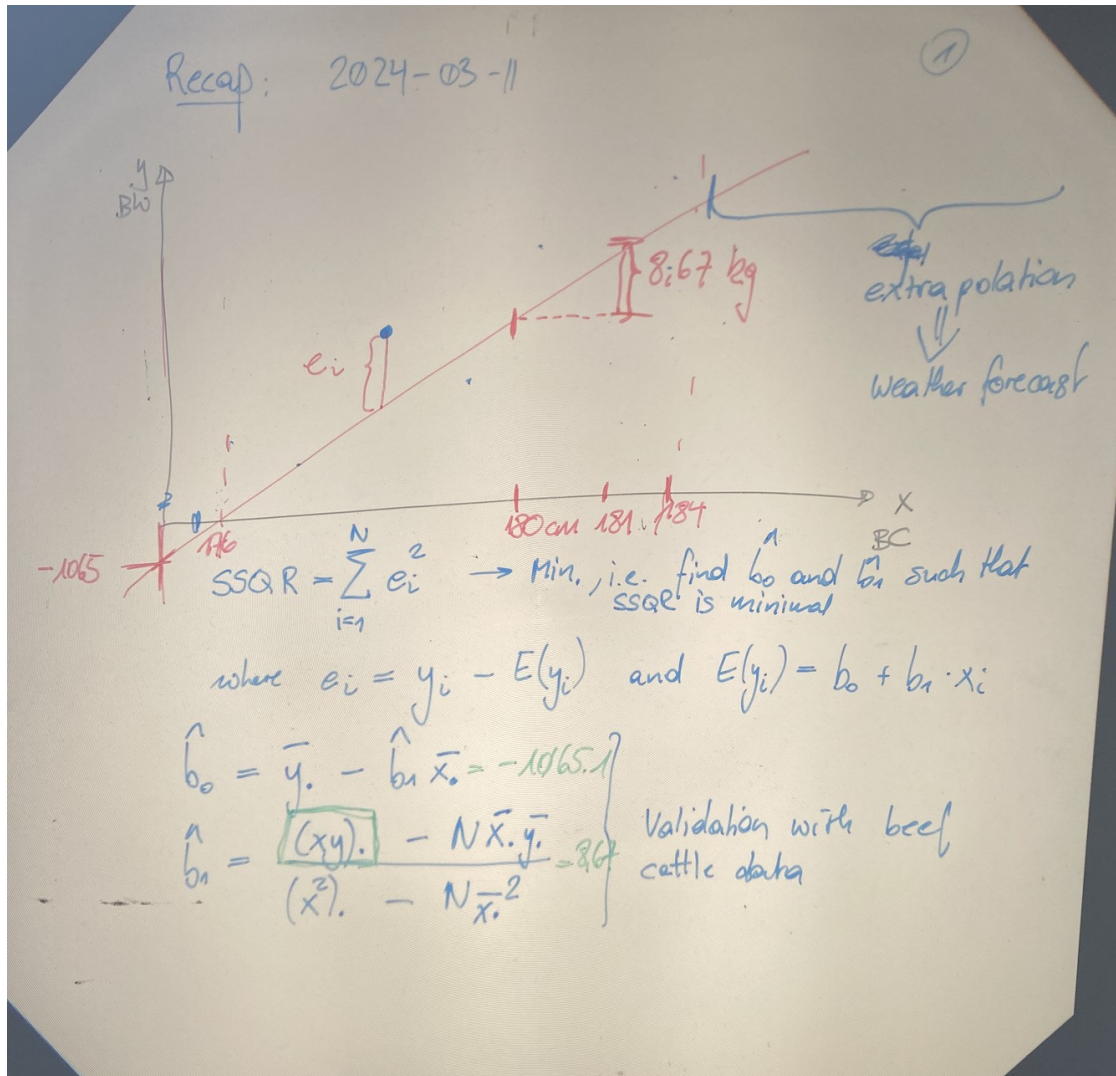


OHP Picture 1



OHP Picture 2

(2)

Simple Linear Regression

□ 1 x-value } fit least-squares (LSQ)
1 y-value

⇒ $\text{lm}()$ in R

□ Beef cattle data:
Linear regression of body weight (y) on
breast circumference (x)

Multiple Linear Regression

□ More than one x-variable, e.g.

- breast circumference (BC) x_1
- height (HEI) x_2

⇒ $E(y_i) = b_0 + b_1 x_{1i} + b_2 x_{2i}$

OHP Picture 3

3

Multiple Linear Regression

□ $SSQR = \sum_{i=1}^N e_i^2$; $e_i = y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}$

□ Goal: Find estimates $\hat{b}_0, \hat{b}_1, \hat{b}_2$ such that SSQR is minimal \Rightarrow Least Squares

$\Rightarrow \frac{\partial SSQR}{\partial b_0} = 0$; $\frac{\partial SSQR}{\partial b_1} = 0$; $\frac{\partial SSQR}{\partial b_2} = 0$

□ Matrix-Vector Notation:

Define three vectors y, e and b

vector $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 471 \\ 463 \\ \vdots \\ 541 \end{bmatrix}$ } observations of response variable

vector $e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$; vector $b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$ } $b_0 \rightarrow$ intercept
 $b_1 \rightarrow$ slope in X_1
 $b_2 \rightarrow$ slope in X_2

OHP Picture 4

④

Matrix $X = \begin{bmatrix} 1 & 176 & 161 \\ 1 & 177 & 121 \\ \vdots & \vdots & \vdots \\ 1 & 184 & 130 \end{bmatrix}$

Flag for intercept
Bread Circumference
Height

□ Model : before $y_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + e_i$

$$\begin{cases} 471 = b_0 + b_1 \cdot 176 + b_2 \cdot 161 + e_1 \\ 463 = b_0 + b_1 \cdot 177 + b_2 \cdot 121 + e_2 \\ \vdots \\ 541 = b_0 + b_1 \cdot 184 + b_2 \cdot 130 + e_{10} \end{cases}$$

New: $y = X \cdot b + e$

$$\begin{bmatrix} 471 \\ 463 \\ \vdots \\ 541 \end{bmatrix} = \begin{bmatrix} 1 & 176 & 161 \\ 1 & 177 & 121 \\ \vdots & \vdots & \vdots \\ 1 & 184 & 130 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{10} \end{bmatrix}$$

Dot-Product

⑤

Least-Squares

Before $e_i = y_i - E(y_i)$
 $= y_i - b_0 - b_1 x_{1i} - b_2 x_{2i}$

- New vector $e = y - X \cdot b$ from $y = Xb + e$
 subtract Xb on both sides

• $SSQR = \sum_{i=1}^N e_i^2 = e_1^2 + e_2^2 + \dots + e_N^2$

New $SSQR = e^T \cdot e$

For a column vector $e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}$ its transpose e^T is defined as the row vector $e^T = [e_1 \ e_2 \ e_3 \ \dots \ e_N]$

By definition of dot-product:

$[e^T \cdot e] = [e_1 \ e_2 \ \dots \ e_N] \cdot \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = e_1^2 + e_2^2 + \dots + e_N^2$

cross product of e , in R: $\text{crossprod}()$

OHP Picture 6

(6)

$$SSQR = e^T \cdot e = (y - Xb)^T \cdot (y - Xb)$$

with $e = y - Xb$

$$= y^T y - y^T Xb - b^T X^T y + \underbrace{b^T X^T X}_{\neq} b$$

□ Least-Squares means, find values of vector b such that SSQR is minimal

□ This is done by taking the "gradient" of SSQR with respect to the vector b :

$$\nabla SSQR = \begin{bmatrix} \frac{\partial SSQR}{\partial b_0} \\ \frac{\partial SSQR}{\partial b_1} \\ \frac{\partial SSQR}{\partial b_2} \end{bmatrix}$$

$$\frac{\partial SSQR}{\partial b} = 0 - y^T X - y^T X + 2 \cancel{b^T X^T X} b^T X^T X$$

$$= -2y^T X + 2b^T X^T X$$

□ Find $\hat{b} \rightarrow 0 = -2y^T X + 2\hat{b}^T X^T X$

OHP Picture 7

(7)

\hat{b} :

$$0 = \underbrace{-2y^T X}_{\leftarrow} + 2\hat{b}^T X^T X$$
$$+ 2y^T X = 2\hat{b}^T X^T X \quad // \text{ } T() \text{ on both sides}$$
$$X^T y = (\hat{b}^T X^T X)^T$$
$$= X^T X \hat{b}$$

\Rightarrow Least-Squares normal equations

$$X^T X \hat{b} = X^T y \quad / \text{ Multiply by } (X^T X)^{-1}$$
$$\underbrace{(X^T X)^{-1}}_I (X^T X) \hat{b} = (X^T X)^{-1} X^T y$$
$$\hat{b} = (X^T X)^{-1} X^T y$$
$$\begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix}$$