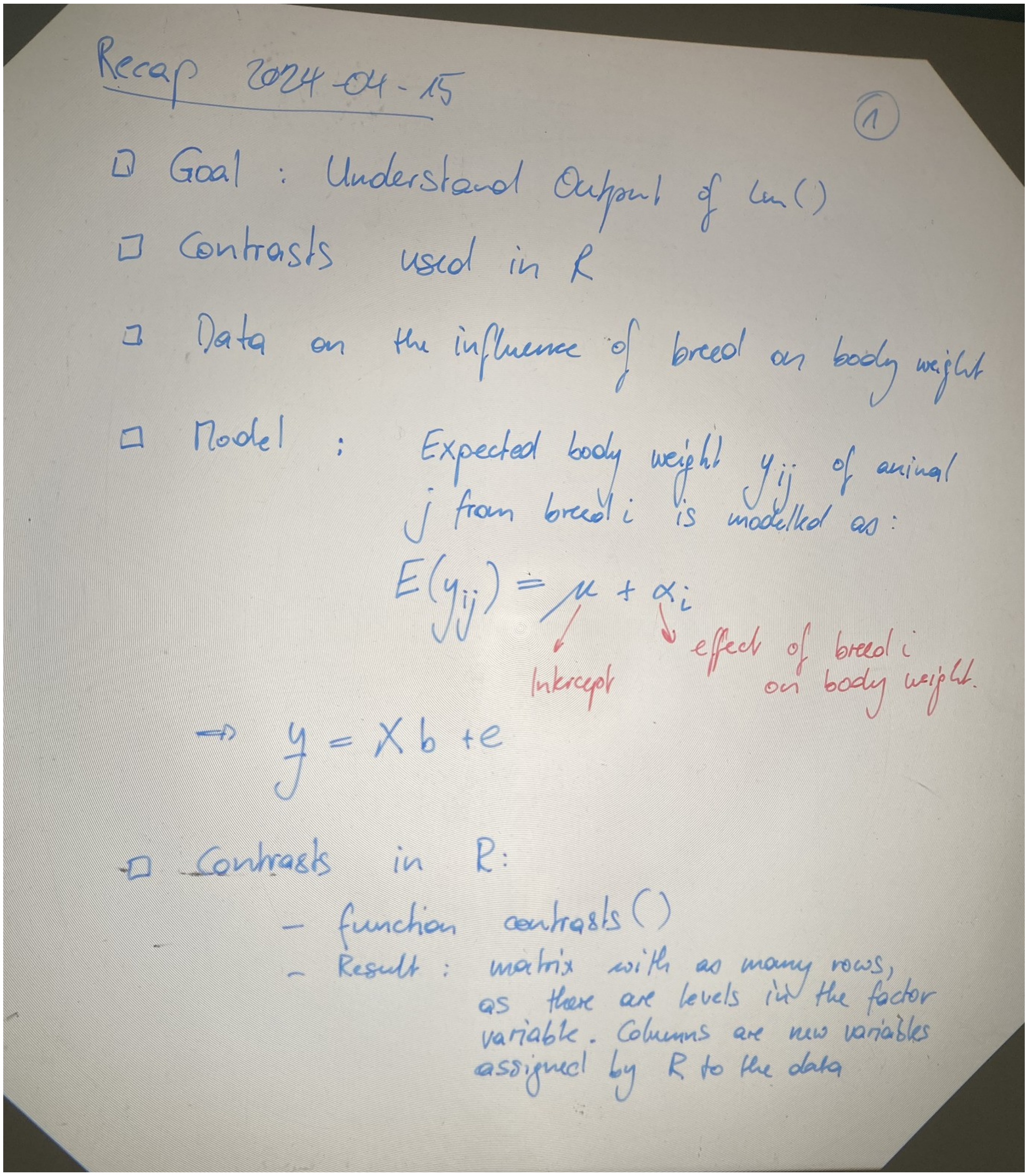
OHP Picture 1



Recap 2024-04-15

1

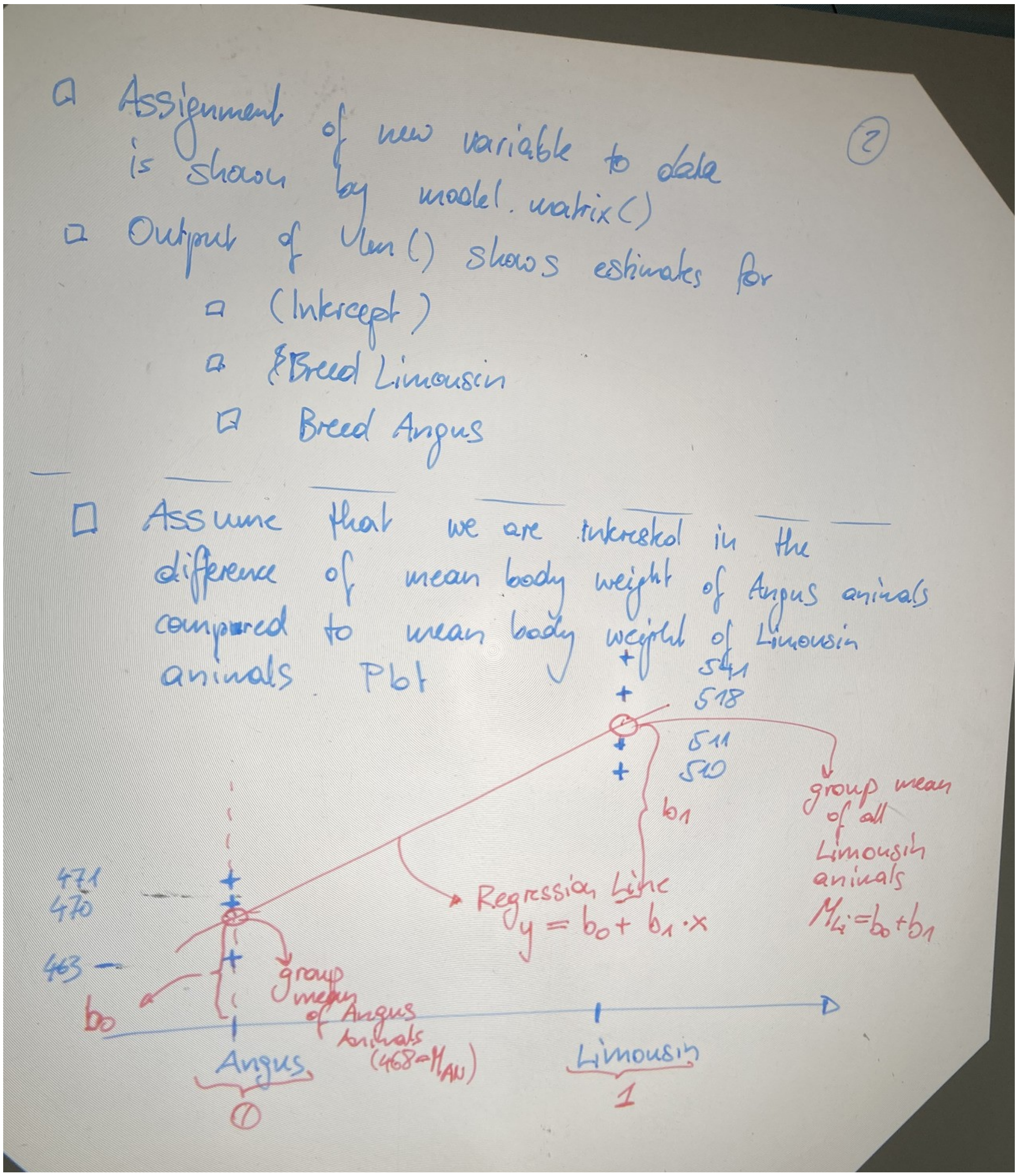
- ☐ Goal: Understand Output of lm()
- ☐ Contrasts used in R
- ☐ Data on the influence of breed on body weight
- ☐ Model: Expected body weight y_{ij} of animal j from breed i is modelled as:

$$E(y_{ij}) = \underbrace{\mu}_{\text{Intercept}} + \underbrace{\alpha_i}_{\text{effect of breed } i \text{ on body weight.}}$$

$$\rightarrow \quad y = Xb + e$$

- ☐ Contrasts in R:
  - function contrasts()
  - Result: matrix with as many rows, as there are levels in the factor variable. Columns are new variables assigned by R to the data

OHP Picture 2



OHP Picture 3

□ Summary:

- Group mean for Angus animals M_{AN}

$$M_{AN} = \frac{1}{N_{AN}} \sum_{i=1}^{N_{AN}} y_{1\cdot}$$

- Group mean M_{Li} for Limousin

$$M_{Li} = \frac{1}{N_{Li}} \sum_{i=1}^{N_{Li}} y_{2\cdot}$$

- From regression of body weight on breed code $\{\text{Angus}: 0 \,;\ \text{Limousin}: 1\}$

$$\left.\begin{aligned} \hat{b}_0 &= M_{AN} \\ \hat{b}_0 + \hat{b}_1 &= M_{Li} \end{aligned}\right\}$$

Define vector $\hat{b} = \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \end{bmatrix}$

$$m = \begin{bmatrix} M_{AN} \\ M_{Li} \end{bmatrix}$$

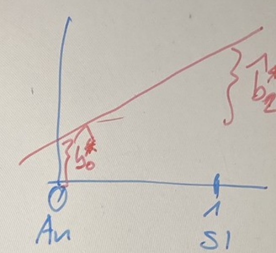
$$m = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \end{bmatrix} = L^T \cdot \hat{b}$$

□ Add Simmental by regression of body weight on breed code $\{\text{Angus}: 0 \,;\ \text{Simmental}: 1\}$

OHP Picture 4

- Difference between body weight of Si and An: (4)
  - + M_{AN} as before
  - + M_{Si} as group mean of body weight of Si animals

$$\begin{bmatrix} M_{AN} \\ M_{Si} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{b}_0^{*} \\ \hat{b}_2^{*} \end{bmatrix}$$



- Combine:

$$m = \underbrace{\begin{bmatrix} M_{AN} \\ M_{Li} \\ M_{Si} \end{bmatrix}}_{\text{known}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{\text{known}} \cdot \underbrace{\begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{bmatrix}}_{\text{unknown} \Rightarrow \text{solve for } \hat{b}} = L^T \cdot \hat{b}$$

$$\underbrace{\left(L^T\right)^{-1}}_{\text{contains estimable functions}} \cdot m = \hat{b}$$

OHP Picture 5

(5)

$$Q^T = (L^T)^{-1} = \begin{array}{c} \begin{array}{ccc} AN & LI & SI \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \rightarrow \text{Intercept} \\ \\ \\ \end{array}$$

$$q_{Li}^T = [\,0 \;\; -1 \;\; 1 \;\; 0\,]$$

$$q_{Si}^T = [\,0 \;\; -1 \;\; 0 \;\; 1\,]$$

$$b^0 = \begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 369.33 \\ 98.667 \\ 150.667 \\ 120.0 \end{bmatrix}$$

$$LI: \; q_{Li}^T \cdot b^0 = 0 \cdot \cancel{\mu} + (-1) \cdot \alpha_1 + 1 \cdot \alpha_2 + 0 \cdot \alpha_3$$
$$= \alpha_2 - \alpha_1 = 52$$

$$SI: \; q_{Si}^T \cdot b^0 = (-1)\alpha_1 + \alpha_3 = \alpha_3 - \alpha_1 = 21.33$$