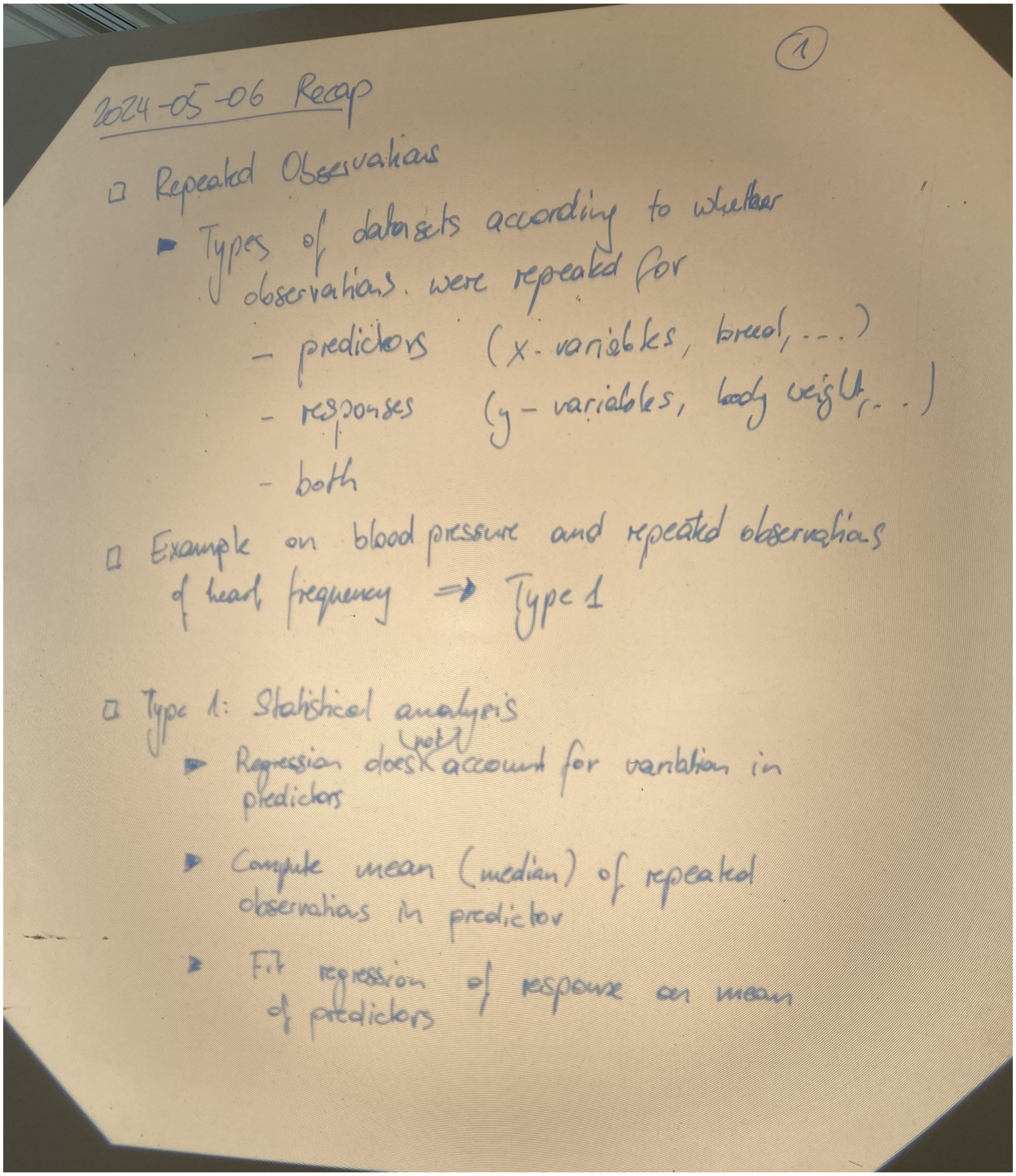
OHP Picture 1

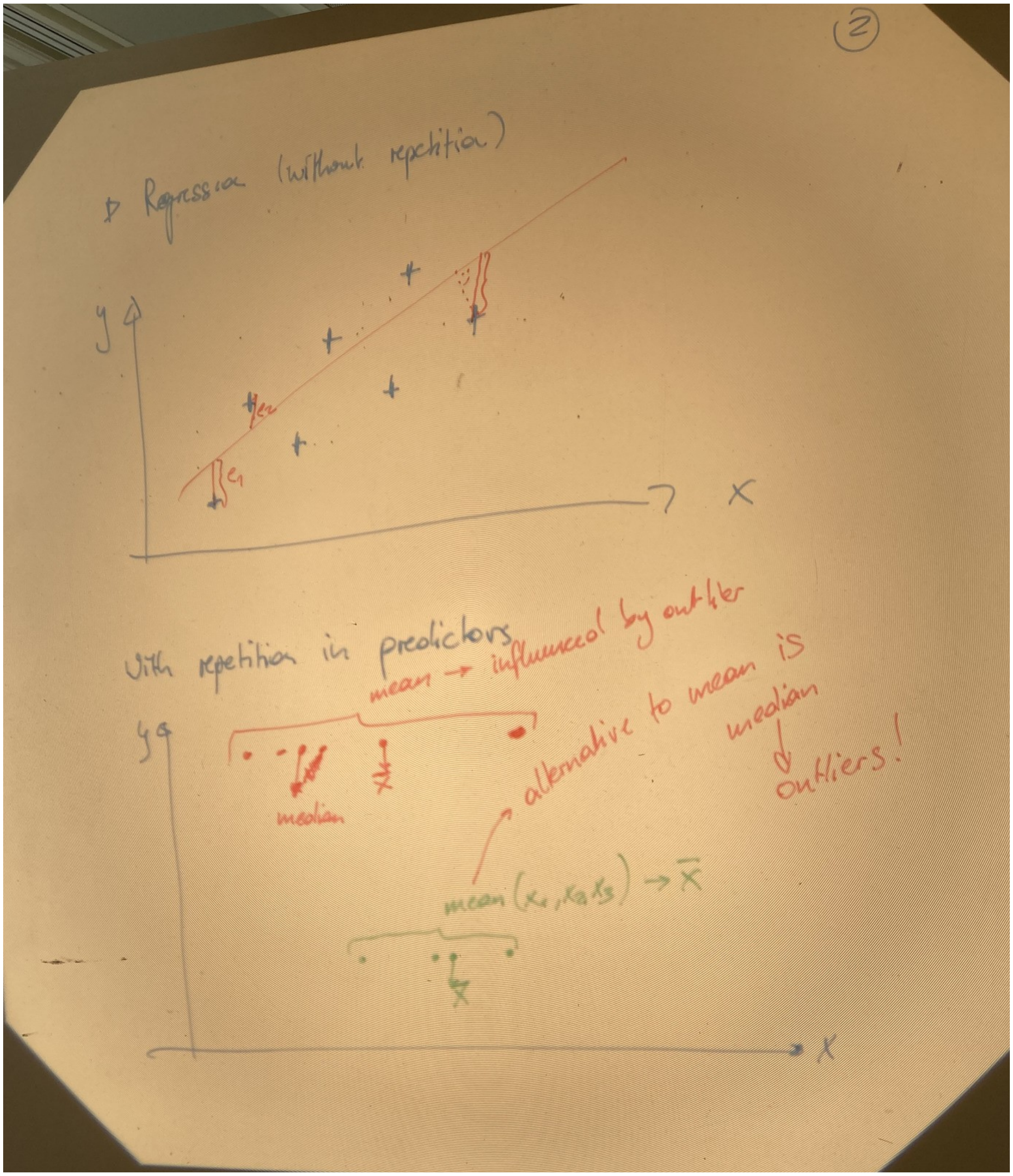


1

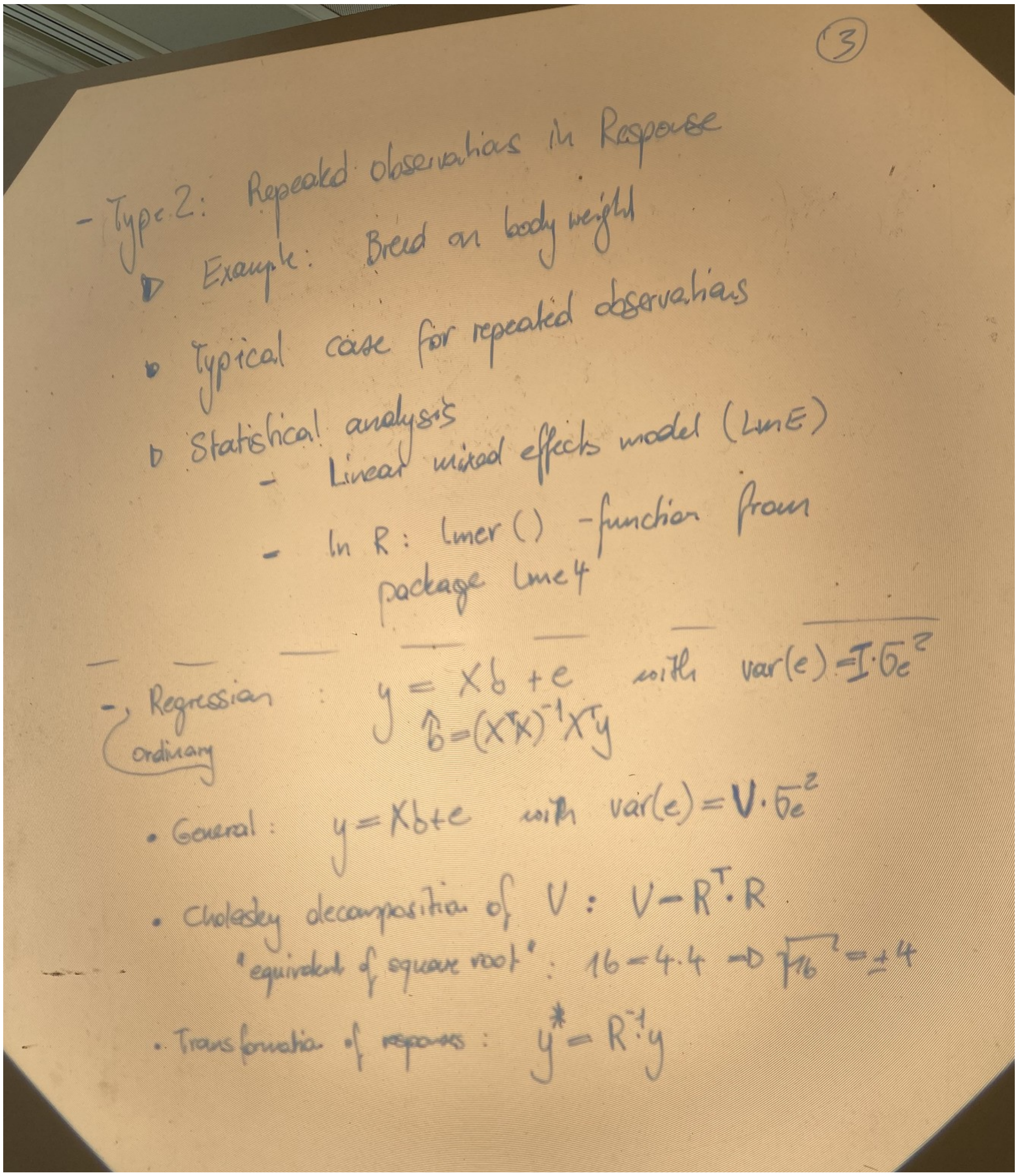
2024-05-06 Recap

- Repeated Observations
  - Types of datasets according to whether observations were repeated for
    - predictors (x-variables, breed, ...)
    - responses (y-variables, body weight, ...)
    - both
- Example on blood pressure and repeated observations of heart frequency → Type 1
- Type 1: Statistical analysis
  - Regression does not account for variation in predictors
  - Compute mean (median) of repeated observations in predictor
  - Fit regression of response on mean of predictors

OHP Picture 2



OHP Picture 3



OHP Picture 4

4

Recap: $V = R^T \cdot R \iff V^{-1} = (R^T R)^{-1}$

$\qquad\qquad\qquad\qquad\qquad = R^{-1}(R^T)^{-1}$

$y^* = R^{-1} \cdot y$

$$y^* = R^{-1} y = R^{-1}(Xb + e) = R^{-1}Xb + R^{-1}e$$

Replace $R^{-1}X$ by X^* and $R^{-1}e$ by e^*

$\Rightarrow \boxed{y^* = X^* b + e^*}$ with $var(e^*) = var(R^{-1}e)$

$$= R^{-1} var(e) (R^{-1})^T$$

$$= R^{-1} var(e) \cdot (R^{-1})^T$$

$$= R^{-1} V \cdot \sigma_e^2 (R^{-1})^T$$

$$= R^{-1} V (R^{-1})^T \cdot \sigma_e^2$$

$$= \underbrace{R^{-1} \cdot R}_{I} \cdot \underbrace{R^T \cdot (R^{-1})^T}_{I} \cdot \sigma_e^2$$

$$= I \cdot \sigma_e^2$$

Ordinary Regression $\}: \hat{b}$

$$\hat{b} = (X^{*T} X^{*})^{-1} X^{*} y^{*}$$

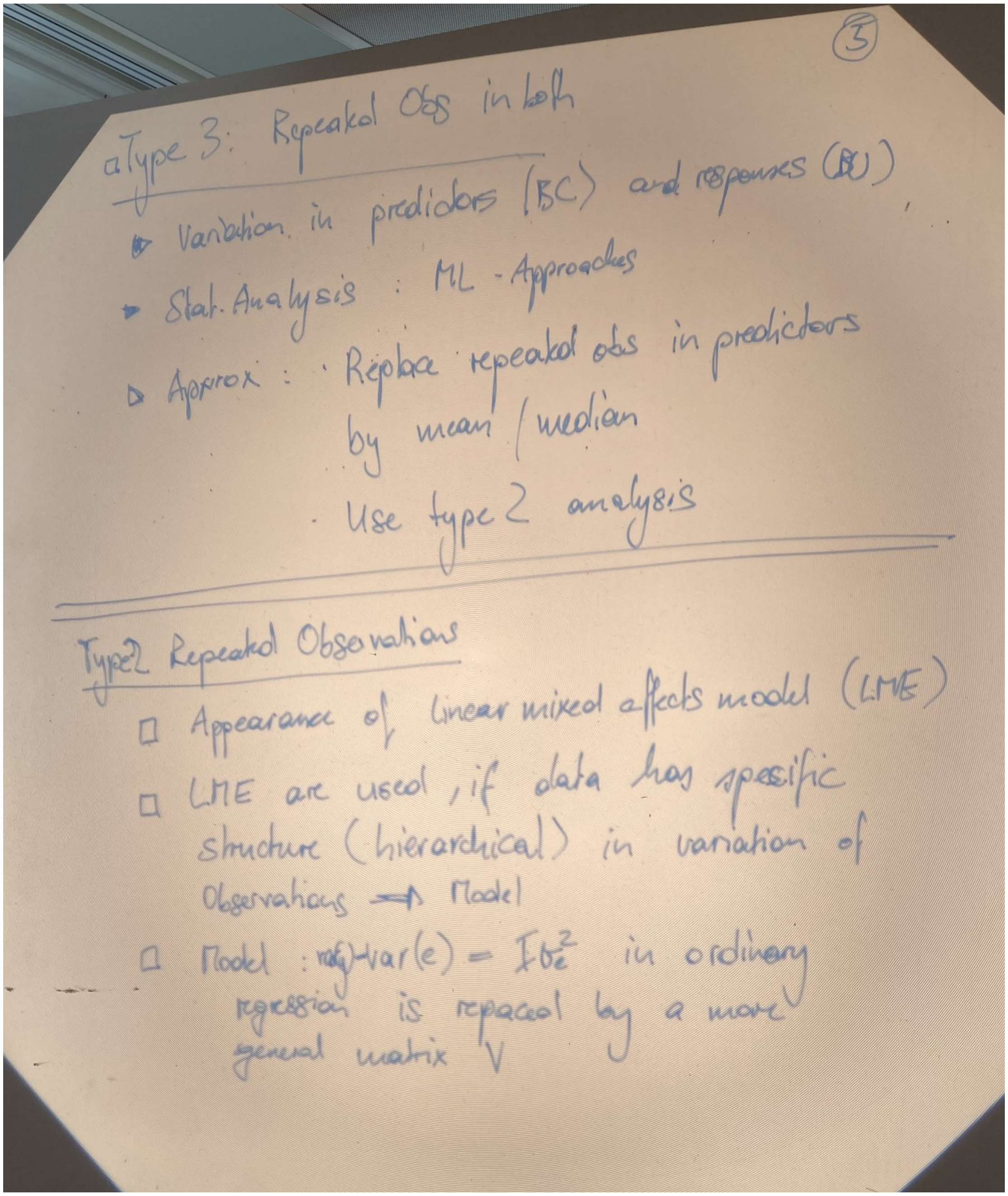
$$= \left([R^{-1}X]^T R^{-1}X\right)^{-1} \cdot [R^{-1}X]^T \cdot R^{-1} y$$

$$= \left(X^T (R^{-1})^T R^{-1} X\right)^{-1} \cdot X^T (R^{-1})^T R^{-1} y$$

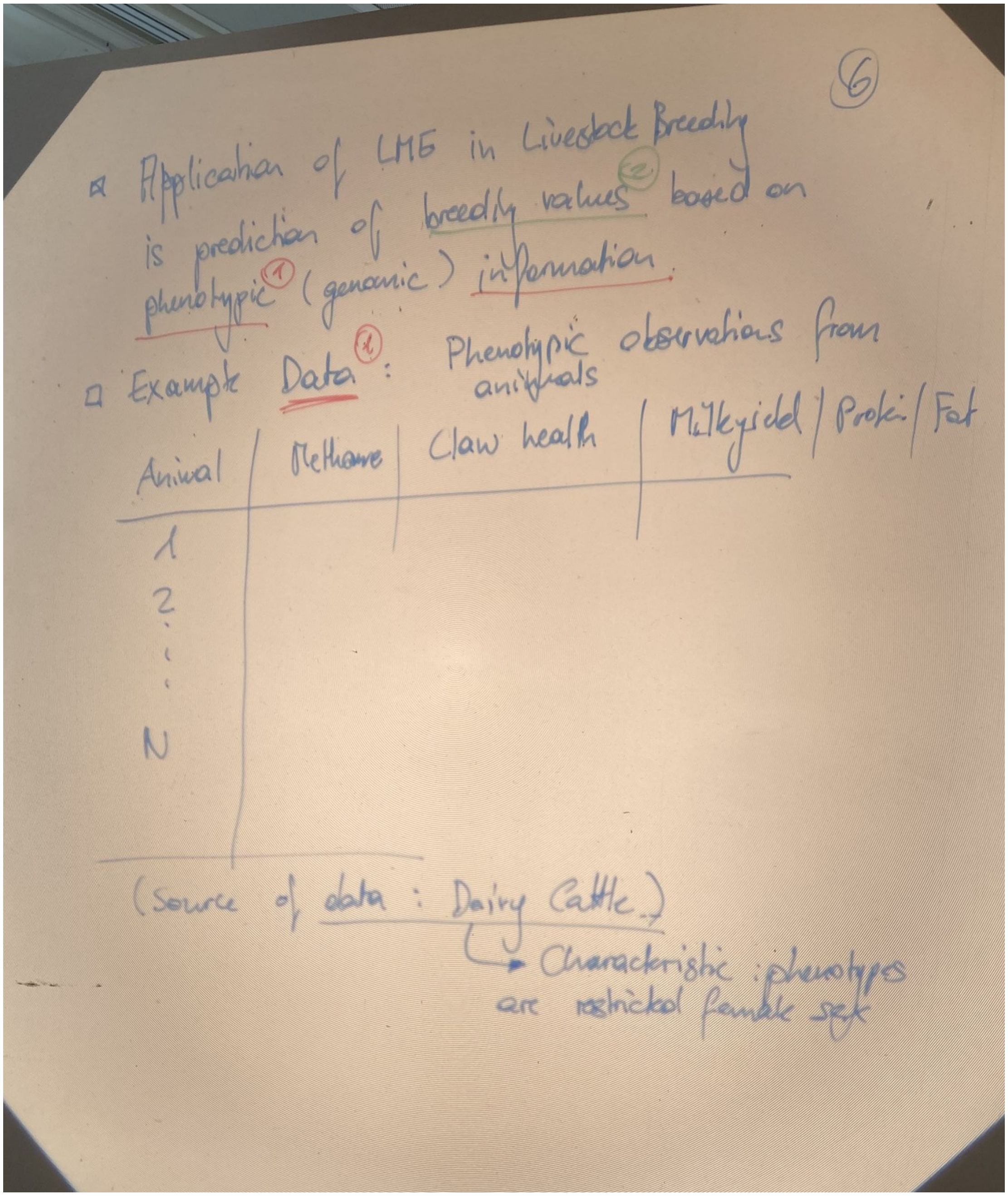
$$= \underline{(X^T V^{-1} X)^{-1} X^T V^{-1} y}$$

Generalized Regression

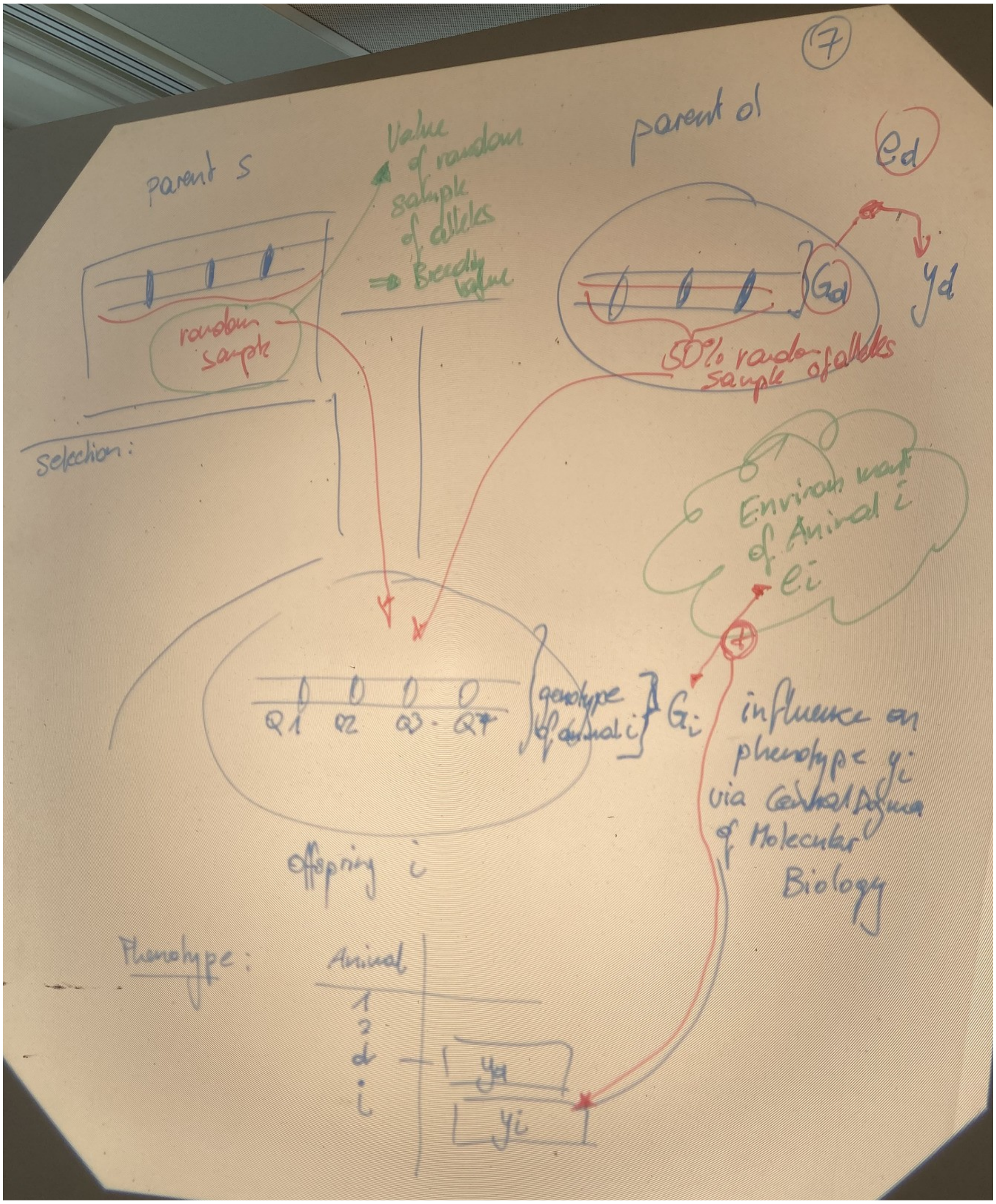
OHP Picture 5



OHP Picture 6



OHP Picture 7



OHP Picture 8