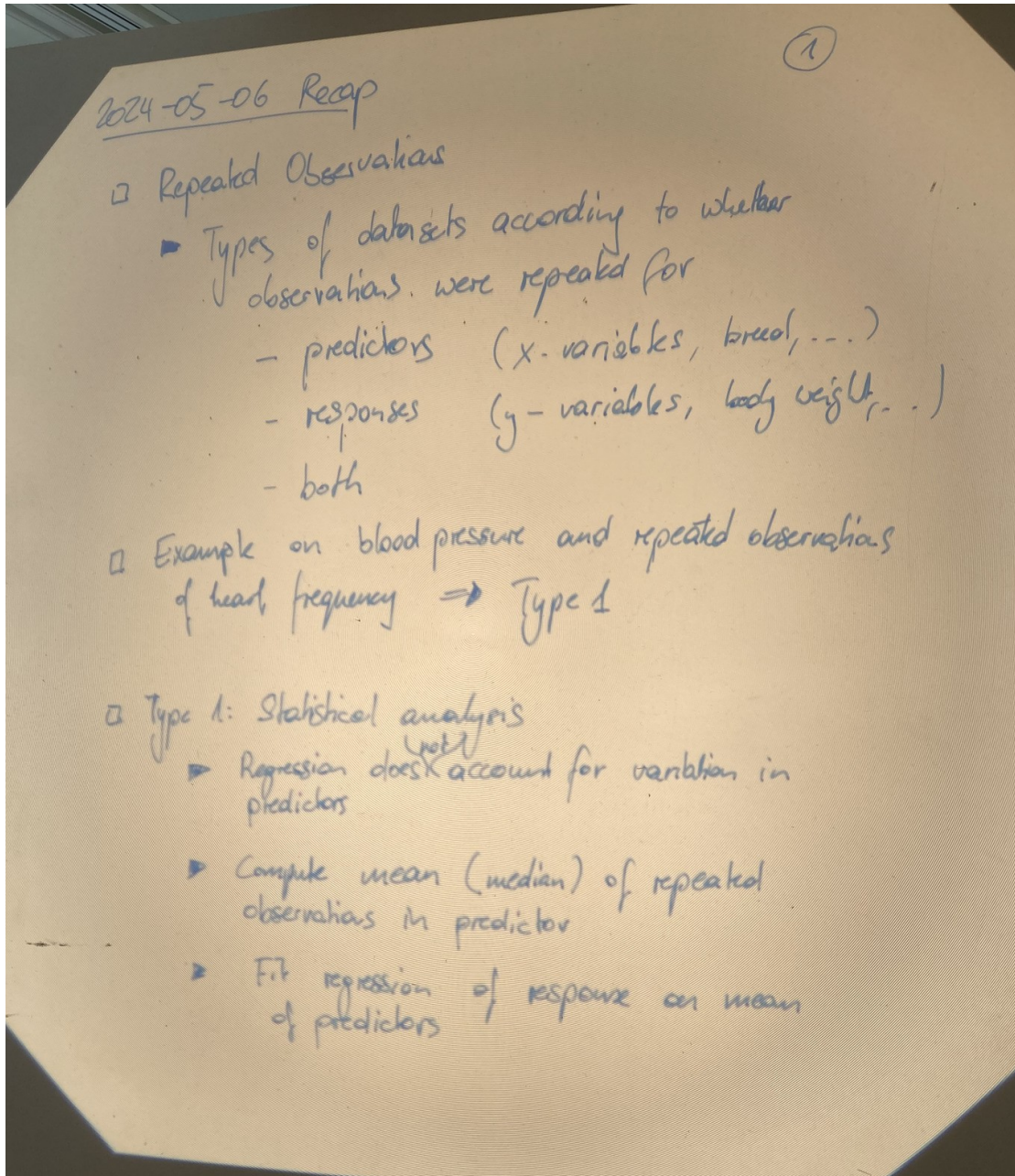
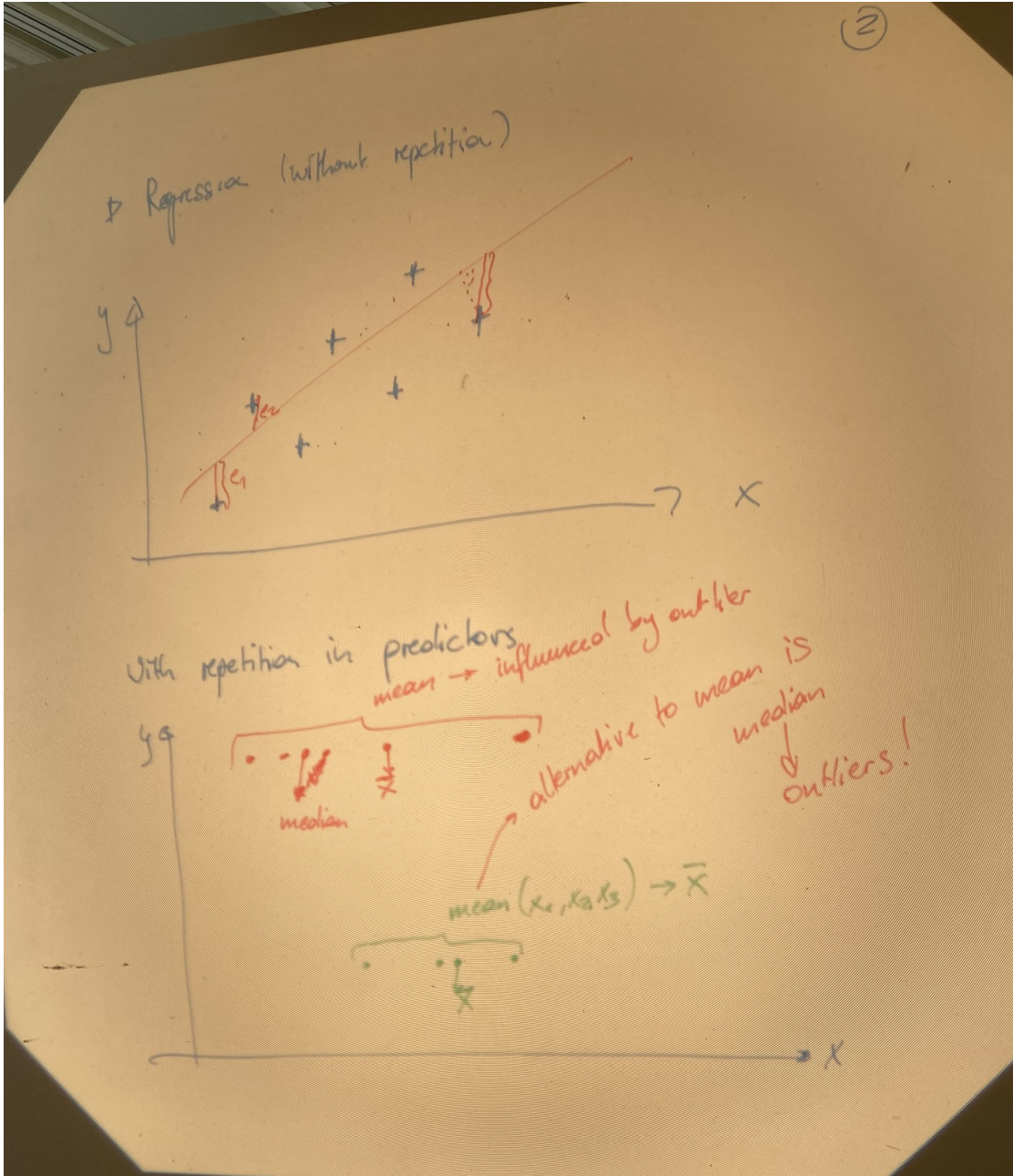


OHP Picture 1



OHP Picture 2



OHP Picture 3

(3)

- Type 2: Repeated observations in Response
 - ▷ Example: Breed on body weight
 - ▷ Typical case for repeated observations
 - ▷ Statistical analysis
 - Linear mixed effects model (LME)
 - In R: `lmer()` - function from package `lme4`

- Regression ordinary

$$y = Xb + e \quad \text{with } \text{var}(e) = I \cdot \sigma_e^2$$
$$\hat{b} = (X^T X)^{-1} X^T y$$

- General: $y = Xb + e$ with $\text{var}(e) = V \cdot \sigma_e^2$
- Cholesky decomposition of V : $V = R^T \cdot R$
"equivalent of square root": $16 = 4 \cdot 4 \rightarrow \sqrt{16} = \pm 4$
- Transformation of response: $y^* = R^{-1} \cdot y$

OHP Picture 4

(4)

Recap: $V = R^T R \Leftrightarrow V^{-1} = (R^T R)^{-1} = R^{-1} (R^T)^{-1}$

$y^* = R^{-1} y$

$y^* = R^{-1} y = R^{-1} (Xb + e) = R^{-1} Xb + R^{-1} e$

Replace $R^{-1} X$ by X^* and $R^{-1} e$ by e^*

$\Rightarrow y^* = X^* b + e^*$ with $\text{var}(e^*) = \text{var}(R^{-1} e)$

↓
Ordinary Regression \hat{b}

$= R^{-1} \text{var}(e) (R^{-1})^T$
 $= R^{-1} \text{var}(e) \cdot (R^{-1})^T$
 $= R^{-1} V \cdot \tilde{\sigma}_e^2 (R^{-1})^T$
 $= R^{-1} V (R^{-1})^T \cdot \tilde{\sigma}_e^2$
 $= \underbrace{R^{-1} \cdot R \cdot R^T}_{I} \cdot \underbrace{(R^{-1})^T}_{I} \cdot \tilde{\sigma}_e^2$
 $= I \cdot \tilde{\sigma}_e^2$

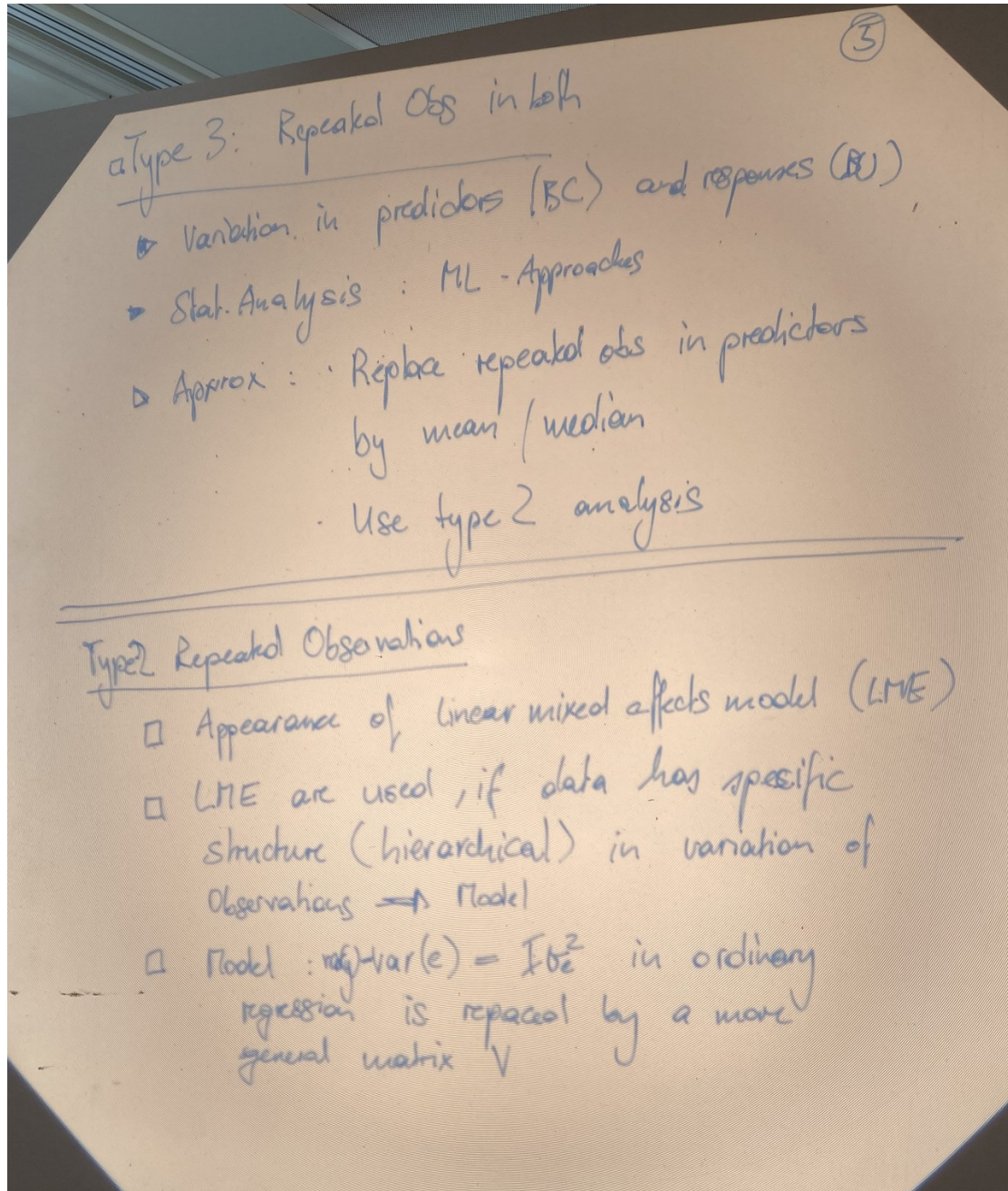
$\hat{b} = (X^{*T} X^*)^{-1} X^{*T} y^*$

$= ([R^T X]^T R^T X)^{-1} [R^T X]^T R^{-1} y$

$= (X^T (R^{-1})^T R^{-1} X)^{-1} X^T (R^{-1})^T R^{-1} y$

$= (X^T V^{-1} X)^{-1} X^T V^{-1} y$

Generalized Regression



OHP Picture 6

⑥

Application of LMS in Livestock Breeding is prediction of breeding values based on phenotypic (genomic) information.

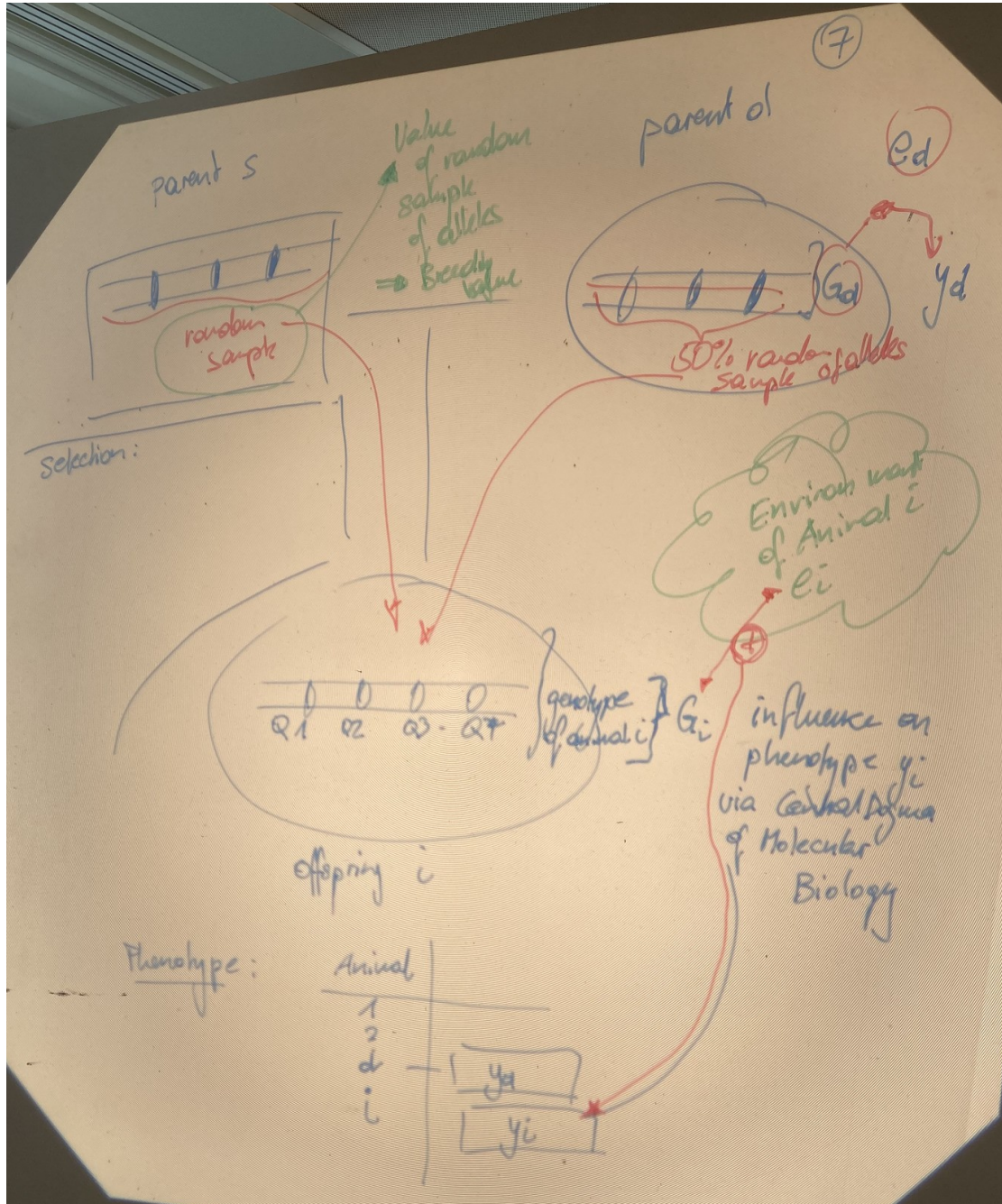
Example Data: Phenotypic observations from animals

Animal	Methane	Claw health	Milk yield	Protein	Fat
1					
2					
⋮					
N					

(Source of data: Dairy Cattle.)

↳ Characteristic: phenotypes are restricted female sex

OHP Picture 7



OHP Picture 8

(8)

Cow	Observations
1	y_1
2	y_2
\vdots	\vdots
i	y_i
\vdots	\vdots
N	y_N

□ Model: Sire Model

$$y_i = \mu + b + s_i + e_i$$

y_i : Observation of cow i

μ : Intercept

s_i : Value of random sample of alleles passed from sire s to offspring i

e_i : random residual

