Interactions

Peter von Rohr

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Definition

- Effect of given predictor variable depends on level or value of other predictor variable
- Examples:
  - Regression of `Body Weight` on `Breast Circumference` is different for different breeds
  - Effect of `Breed` on `Body Weight` is different for different male and female animals

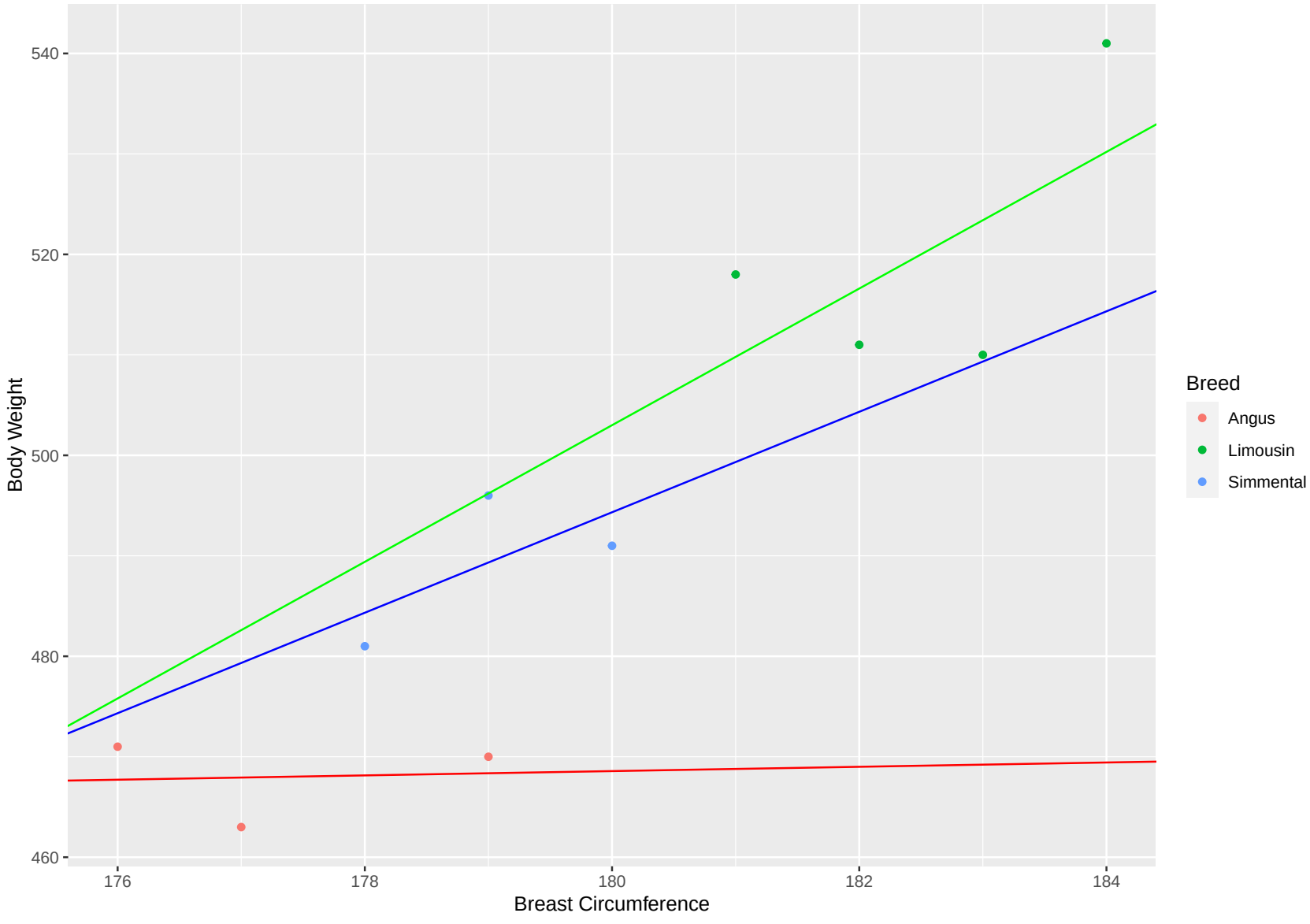
Types of Interactions

- continuous by continuous
- continuous by categorical
- categorical by categorical

Continuous by Categorical

- In a model, expected value of response depends on regression variable plus discrete factor
- Example: Regression of `Body Weight` on `Breast Circumference` plus the factor `Breed`
- Interaction is present, if regression of `Body Weight` on `Breast Circumference` is different for different breeds

Interaction Plot



Interaction Model

- Start with model without interactions

$$y_i = b_0 + b_1 \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

- Assume linear relationship of b_1 with `Breed`

$$b_1 = a + b_4 \times BrLi + b_5 \times BrSi$$

- Insert

$$y_i = b_0 + (a + b_4 \times BrLi + b_5 \times BrSi) \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

- Simplify

$$\begin{aligned} y_i = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i \\ + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i + e_i \end{aligned}$$

Continuous by Continuous

- Similar to continuous by categorical
- No interaction

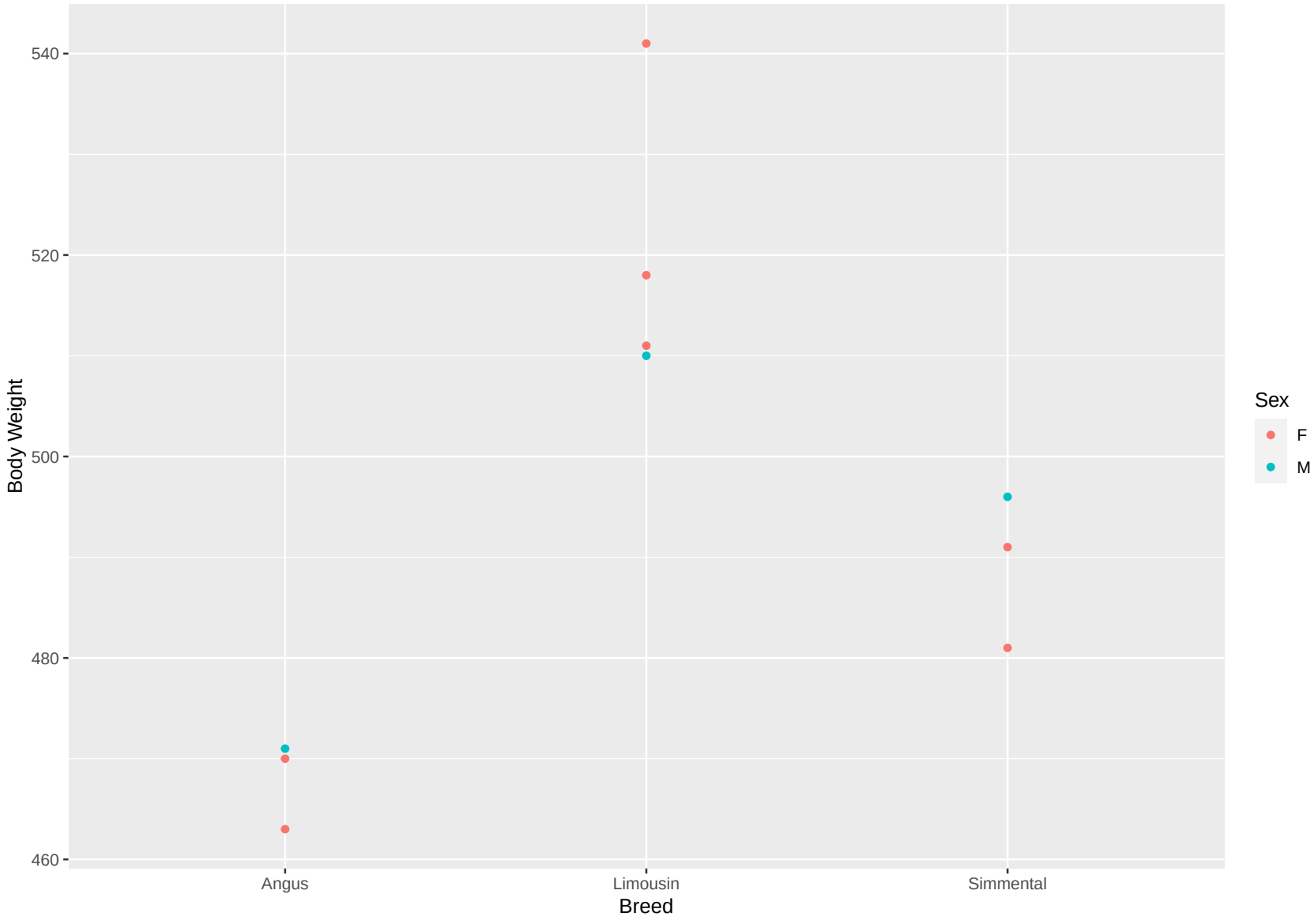
$$y_0 = b_0 + b_1 \times BC_i + b_2 \times HE_i + e_i$$

- Interaction by dependence of one regression coefficient on other coefficient

$$b_1 = b_3 + b_4 \times HE_i$$

$$y_0 = b_0 + b_2 \times HE_i + b_3 \times BC_i + b_4 \times HE_i \times BC_i + e_i$$

Categorical by Categorical



Model Matrix

```
##    (Intercept) BreedLimousin BreedSimmental SexM BreedLimousin:SexM
## 1            1             0               0    1                  0
## 2            1             0               0    0                  0
## 3            1             0               0    0                  0
## 4            1             1               0    0                  0
## 5            1             1               0    0                  0
## 6            1             1               0    1                  1
## 7            1             1               0    0                  0
## 8            1             0               1    0                  0
## 9            1             0               1    1                  0
## 10           1             0               1    0                  0
##    BreedSimmental:SexM
## 1                    0
## 2                    0
## 3                    0
## 4                    0
## 5                    0
## 6                    0
## 7                    0
## 8                    0
## 9                    1
## 10                   0
## attr(,"assign")
## [1] 0 1 1 2 3 3
## attr(,"contrasts")
## attr(,"contrasts")$Breed
## [1] "contr.treatment"
##
## attr(,"contrasts")$Sex
## [1] "contr.treatment"
```

Summary

```
##
## Call:
## lm(formula = `Body Weight` ~ Breed * Sex, data = tbl_flem_bw_br_sex)
##
## Residuals:
##          1          2          3          4          5          6          7
##  3.726e-15 -3.500e+00  3.500e+00 -5.333e+00 -1.233e+01 -1.703e-15  1.767e+01
##          8          9         10
## -5.000e+00 -6.458e-16  5.000e+00
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)           466.50       8.42  55.404 6.35e-07 ***
## BreedLimousin          56.83      10.87   5.228  0.00639 **
## BreedSimmental         19.50      11.91   1.638  0.17685
## SexM                    4.50      14.58   0.309  0.77306
## BreedLimousin:SexM    -17.83      20.04  -0.890  0.42389
## BreedSimmental:SexM     5.50      20.62   0.267  0.80291
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.91 on 4 degrees of freedom
## Multiple R-squared:  0.8981, Adjusted R-squared:  0.7706
## F-statistic: 7.048 on 5 and 4 DF,  p-value: 0.04092
```