

Linear Regression

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Goal

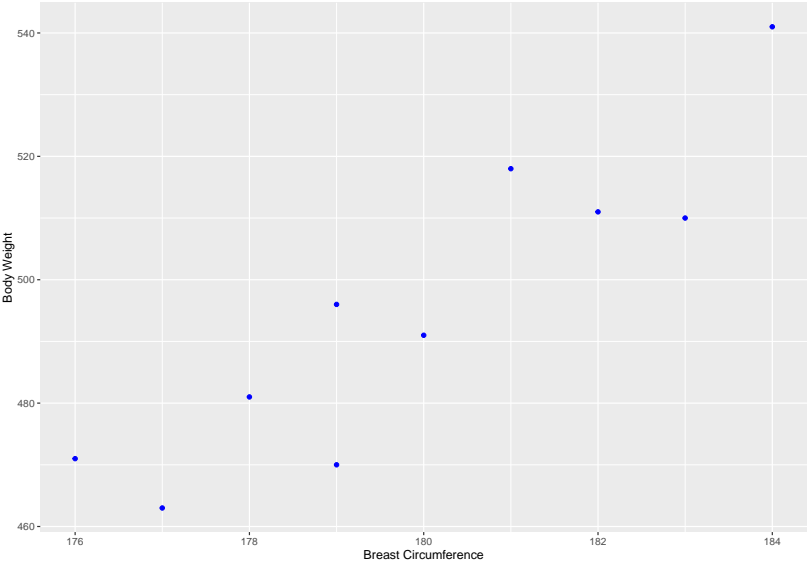
Assessment of relationship between

- ▶ a given variable (response) and
- ▶ other measurements or observations (predictors) on the same animal

Example

Animal	Breast Circumference	Body Weight
1	176	471
2	177	463
3	178	481
4	179	470
5	179	496
6	180	491
7	181	518
8	182	511
9	183	510
10	184	541

Diagram

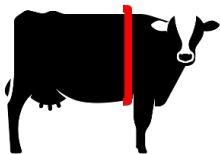


Observations

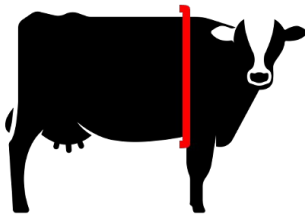
- ▶ relationship between breast circumference and body weight: heavier animals tend to have larger values for breast circumference
- ▶ same relationship across whole range → **linear** relationship

Regression Model

- ▶ quantify relationship between body weight and breast circumference
- ▶ practical application: measure band for animals



Created by Agniraj Chatterji
from Noun Project



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Model Building

- ▶ expected body weight ($E(y)$ in kg) based on an observed value of x cm for breast circumference

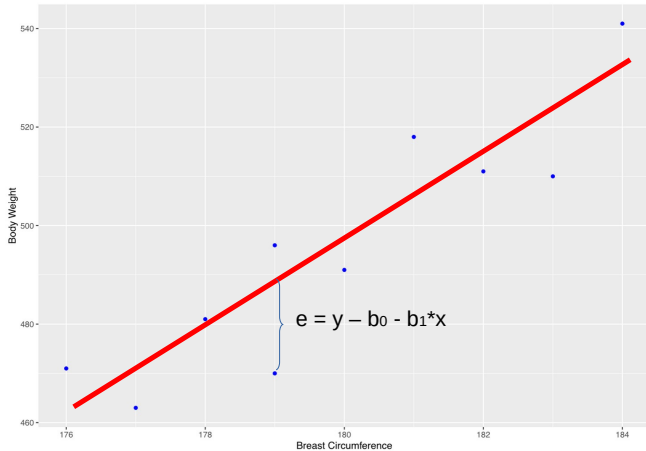
$$E(y) = b_0 + b_1 * x$$

- ▶ b_0 and b_1 are unknown parameters of the model
- ▶ model is linear function of parameters \rightarrow linear model

Parameter Estimation

- ▶ How to find values for b_0 and b_1
- ▶ several techniques available: start with Least Squares

Least Squares



Estimators

Find values \hat{b}_0 and \hat{b}_1 such that

$$\mathbf{e}^T \mathbf{e} = \sum_{i=1}^N e_i^2 = \sum_{i=1}^N [y_i - E(e_i)]^2 = \sum_{i=1}^N [y_i - b_0 - b_1 * x_i]^2$$

is minimal

Minimization

$$\begin{aligned}\frac{\partial \mathbf{e}^T \mathbf{e}}{\partial b_0} &= -2 \sum_{i=1}^N [y_i - b_0 - b_1 x_i] \\ &= -2 \left[\sum_{i=1}^N y_i - N b_0 - b_1 \sum_{i=1}^N x_i \right]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{e}^T \mathbf{e}}{\partial b_1} &= -2 \sum_{i=1}^N x_i [y_i - b_0 - b_1 x_i] \\ &= -2 \left[\sum_{i=1}^N x_i y_i - b_0 \sum_{i=1}^N x_i - b_1 \sum_{i=1}^N x_i^2 \right]\end{aligned}$$

Notation

$$x_{\cdot} = \sum_{i=1}^N x_i$$

$$y_{\cdot} = \sum_{i=1}^N y_i$$

$$(x^2)_{\cdot} = \sum_{i=1}^N x_i^2$$

$$(xy)_{\cdot} = \sum_{i=1}^N x_i y_i$$

$$\bar{x}_{\cdot} = \frac{x_{\cdot}}{N}$$

$$\bar{y}_{\cdot} = \frac{y_{\cdot}}{N}$$

Solutions

$$\hat{b}_0 = \bar{y} - \hat{b}_1 \bar{x}.$$

and

$$\hat{b}_1 = \frac{(xy) - N\bar{x}\bar{y}}{(x^2) - N\bar{x}^2}$$