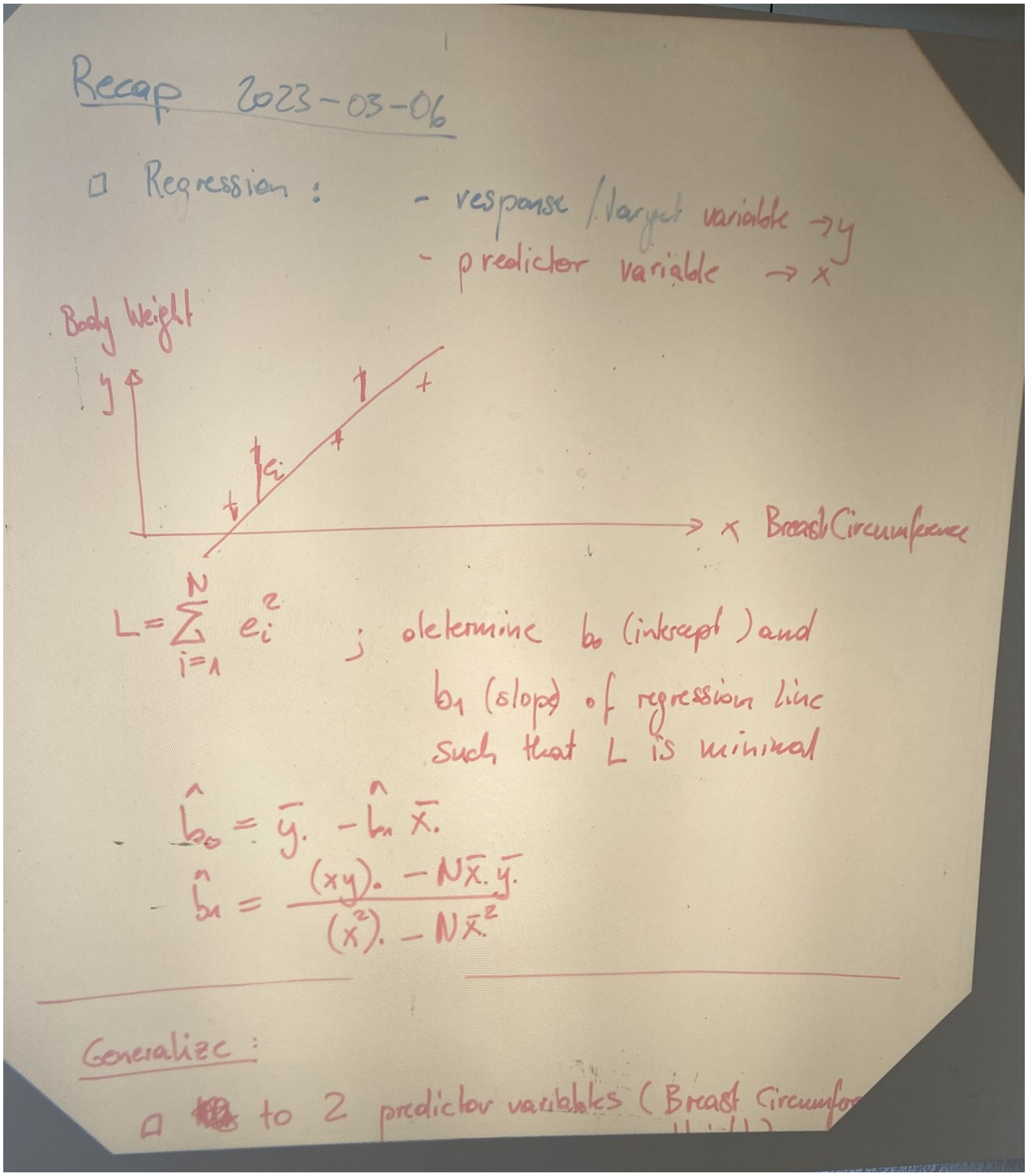
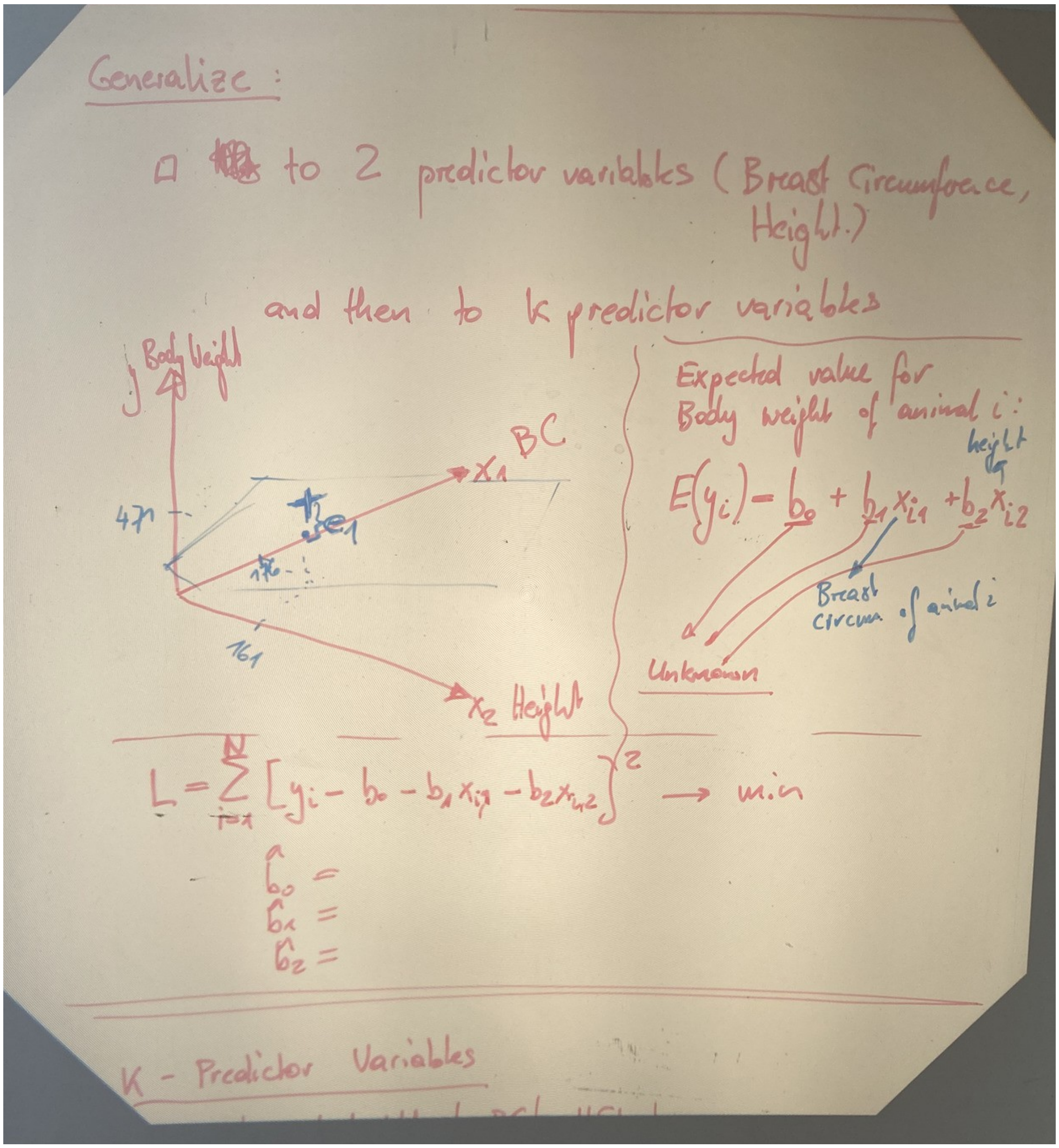
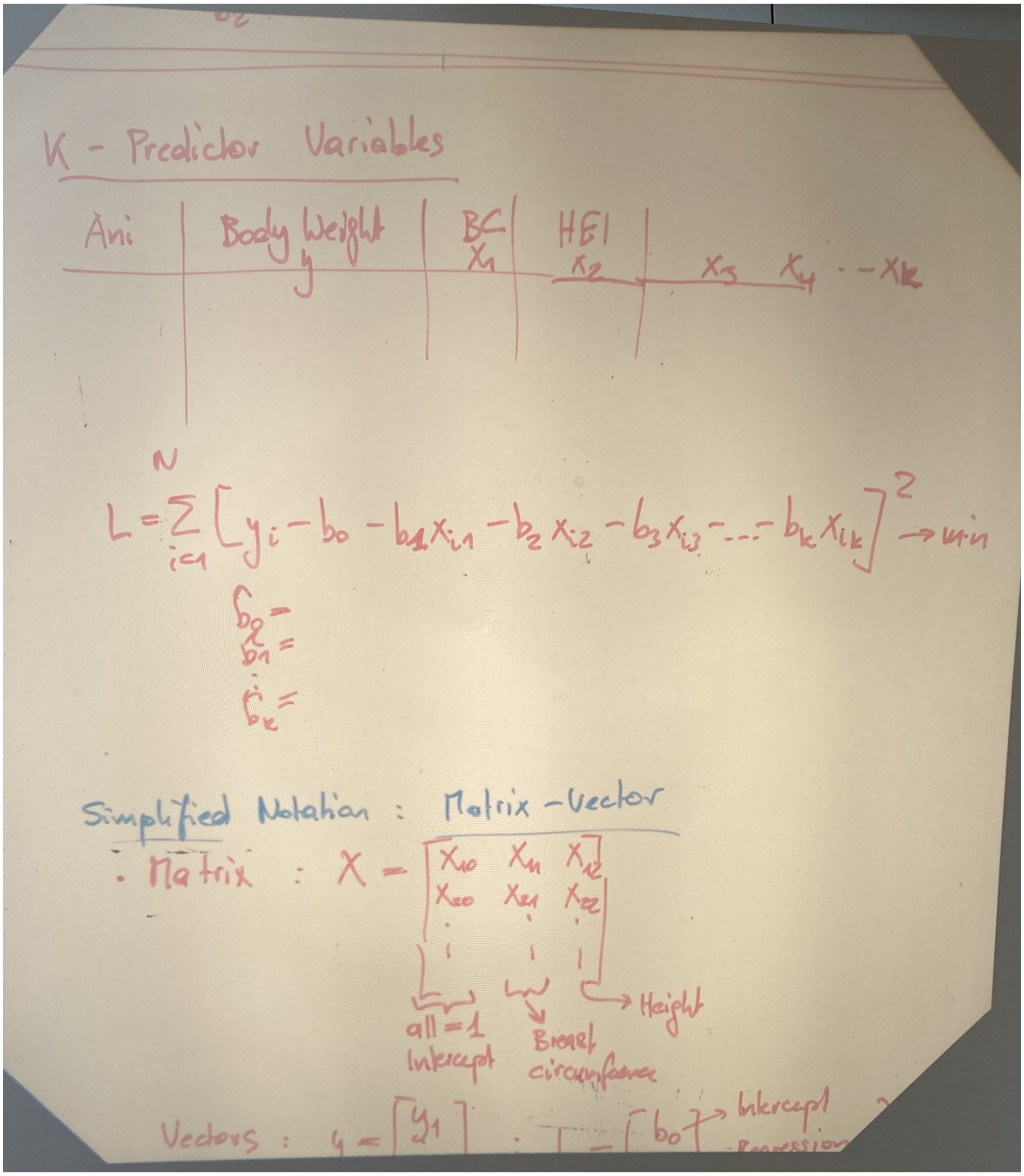
OHP Picture 1



OHP Picture 2



OHP Picture 3



OHP Picture 4

Simplified Notation: Matrix-Vector

• Matrix:

$$X = \begin{bmatrix} x_{10} & x_{11} & x_{12} \\ x_{20} & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\begin{array}{ccc} \text{all} = 1 & \text{Breast} & \text{Height} \\ \text{Intercept} & \text{circumference} & \end{array}$$

Vectors:

$$\underbrace{y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}}_{\text{Body weight}} ; \quad b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \begin{array}{l} \rightarrow \text{Intercept} \\ \rightarrow \text{Regression coefficient for BC} \\ \rightarrow \text{Regression coefficient for Height} \end{array} \Bigg\} \text{unknown}$$

$$e = \begin{bmatrix} e_1 \\ \vdots \\ e_N \end{bmatrix}$$

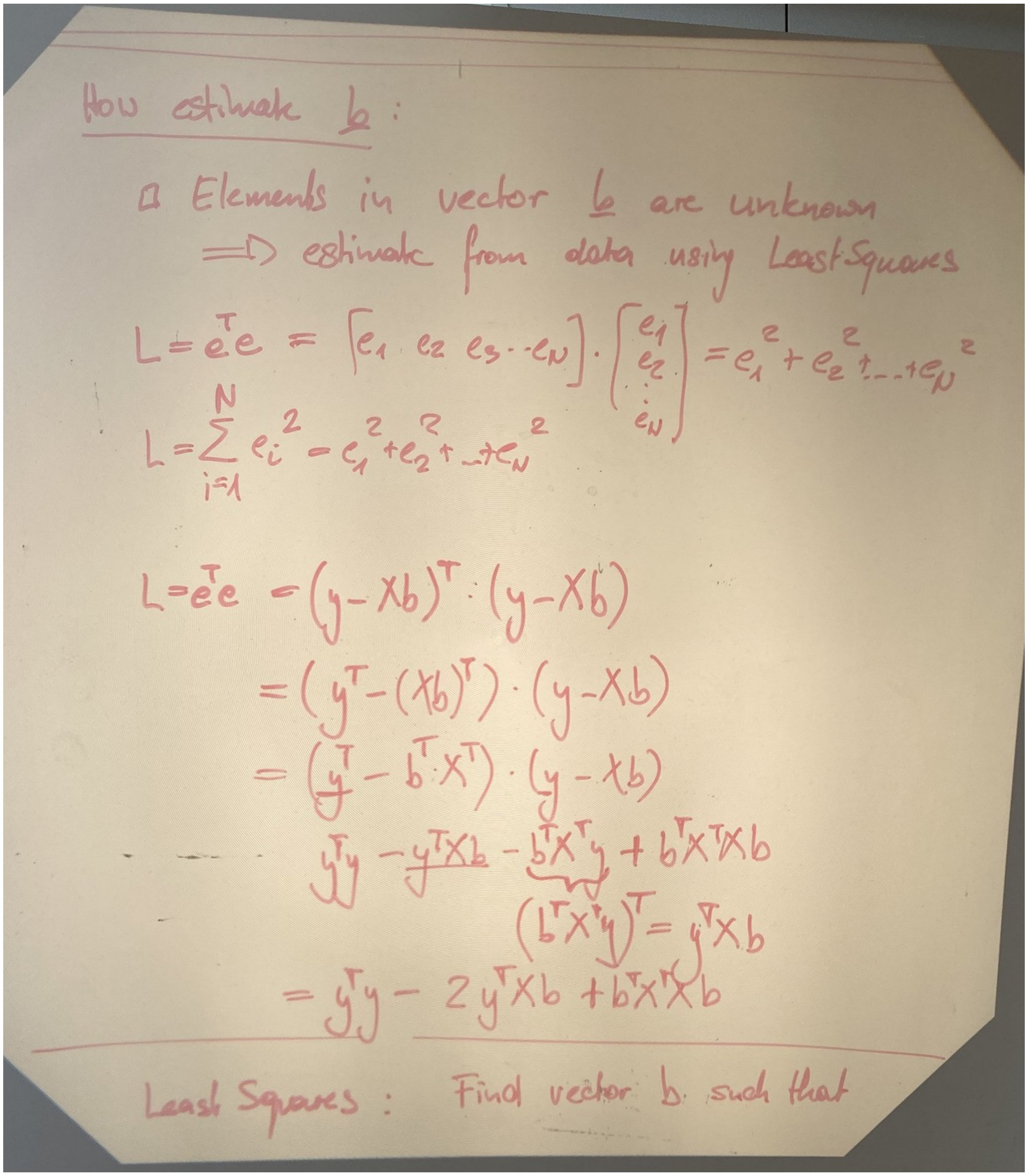
$$\begin{cases} E(y_1) = b_0 + b_1 x_{11} + b_2 x_{21} \\ E(y_2) = b_0 + b_1 x_{12} + b_2 x_{22} \\ \vdots \\ E(y_N) \end{cases}$$

$$E(y_i) = b_0 + b_1 x_{1i} + b_2 x_{2i}$$

$$E(y) = Xb$$

$$y = Xb + e \Rightarrow e = y - Xb$$

OHP Picture 5



How estimate $\underline{b}$:

□ Elements in vector $\underline{b}$ are unknown

$\Longrightarrow$ estimate from data using Least-Squares

$$L = e^T e = [e_1\ e_2\ e_3 \cdots e_N] \cdot \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = e_1^2 + e_2^2 + \dots + e_N^2$$

$$L = \sum_{i=1}^{N} e_i^2 = e_1^2 + e_2^2 + \dots + e_N^2$$

$$L = e^T e = (y - Xb)^T \cdot (y - Xb)$$

$$= (y^T - (Xb)^T) \cdot (y - Xb)$$

$$= (y^T - b^T X^T) \cdot (y - Xb)$$

$$y^T y - y^T X b - \underbrace{b^T X^T y}_{(b^T X^T y)^T = y^T X b} + b^T X^T X b$$

$$= y^T y - 2 y^T X b + b^T X^T X b$$

Least Squares: Find vector b such that

OHP Picture 6

Least Squares: Find vector b such that

$$\underbrace{\frac{\partial L}{\partial b_0};\ \frac{\partial L}{\partial b_1};\ \frac{\partial L}{\partial b_2};\ \cdots\ \frac{\partial L}{\partial b_k}}_{\text{Compute gradient of } L \text{ with respect to } b} \text{ are all } \emptyset$$

$$\frac{\partial L}{\partial b} = \begin{bmatrix} \frac{\partial L}{\partial b_0} \\ \frac{\partial L}{\partial b_1} \\ \vdots \\ \frac{\partial L}{\partial b_k} \end{bmatrix}$$

$$L = y^T y - 2y^T X b + b^T X^T X b \qquad \frac{\partial x^n}{\partial x} = n x^{n-1}$$

$$\frac{\partial L}{\partial b} = \emptyset - 2y^T X + 2b^T X^T X$$

Find b such that $\frac{\partial L}{\partial b} = \emptyset$

$$\Rightarrow -2y^T X + 2\hat{b}^T X^T X = 0$$

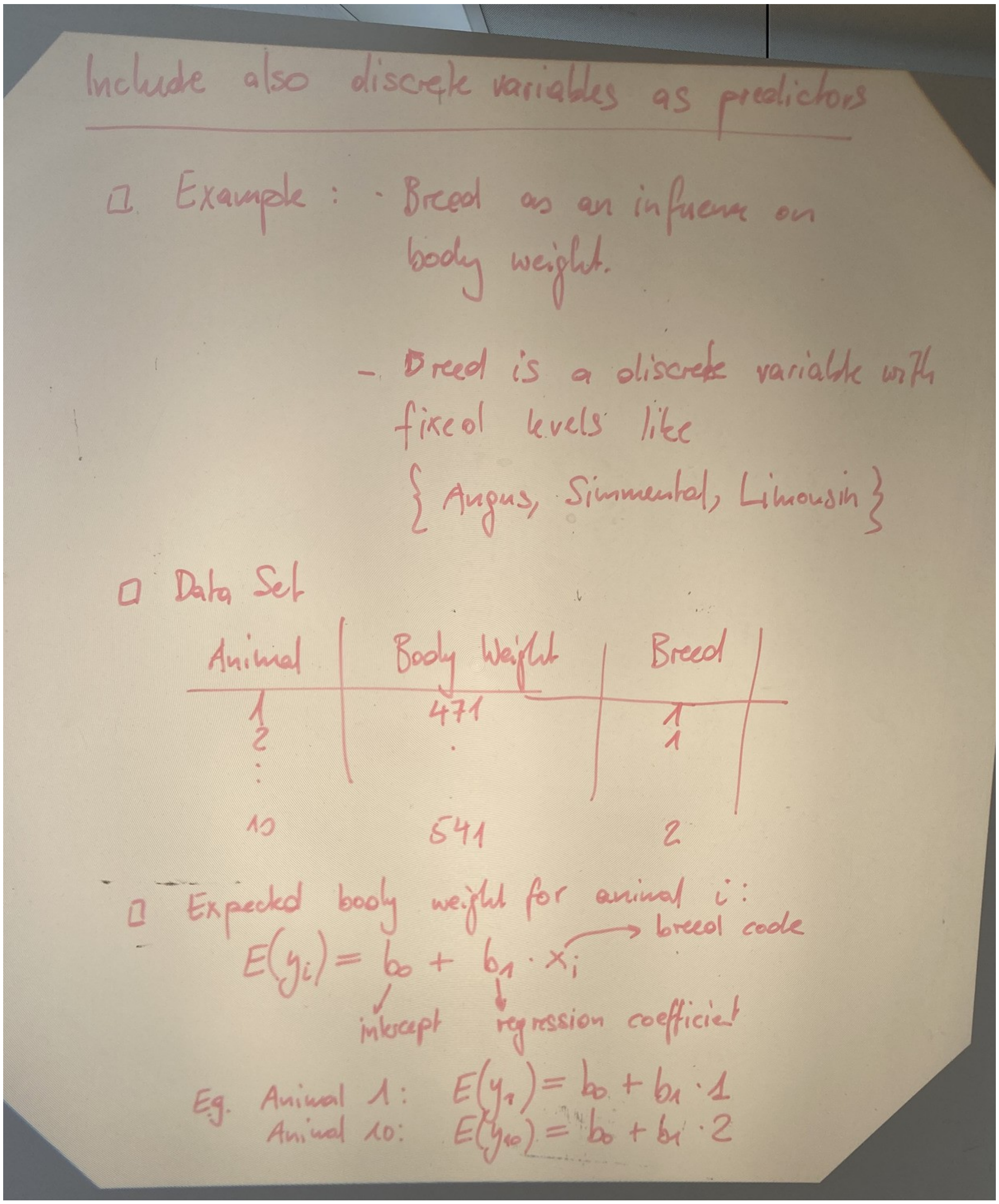
$$\hat{b}^T X^T X = y^T X$$

$$(X^T X)\hat{b} = X^T y \Rightarrow \text{Normal Equations}$$

Given that $(X^T X)$ can be inverted:

$$\hat{b} = (X^T X)^{-1} X^T y$$

OHP Picture 7



Include also discrete variables as predictors

- Example:
  - Breed as an influence on body weight.
  - Breed is a discrete variable with fixed levels like {Angus, Simmental, Limousin}
- Data Set

Animal	Body Weight	Breed
1	471	1
2	.	1
:		
10	541	2

- Expected body weight for animal i:

$$E(y_i) = \underbrace{b_0}_{\text{intercept}} + \underbrace{b_1}_{\text{regression coefficient}} \cdot \underbrace{x_i}_{\text{breed code}}$$

Eg. Animal 1: $E(y_1) = b_0 + b_1 \cdot 1$
Animal 10: $E(y_{10}) = b_0 + b_1 \cdot 2$