

## OHP Picture 1

Recap:

A Goal: Understand of summary ( $\ln(1)$ )

with factors and levels, such Breed

□ Assign separate effects to each level (different breeds)

⇒ Matrix  $X$  in model  $y = Xb + e$   
has not full column rank

⇒ No single solution to the least squares normal equation

$$X^T X b^0 = X^T y$$

□ A solution is given by  $b^0 = (X^T X)^- X^T y$   
 $= G X^T y$

with  $G = (X^T X)^-$

Problems:

$b^0$  is not unique, because

$$\tilde{b} = GX^T y + (G(X^T X) - I)z$$

is also a solution to  $X^T X b = X^T y$

~~$$X^T X GX^T y = X^T X y$$~~

$$X^T X (GX^T y + (G(X^T X) - I)z)$$

$$X^T X GX^T y + X^T X G(X^T X)z - X^T X \cdot I z$$

$$= X^T X GX^T y + \underbrace{X^T X z - X^T X z}$$

$$= X^T X \underbrace{GX^T y}_{b^0} = X^T X b^0 = X^T y$$


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A solution  $b^0$  is not an estimate for the effects vector  $b$ ; because

$$E(b^0) = E(GX^T y) = GX^T E(y)$$

$$= \underline{GX^T X} b \neq b$$

In Regression:  $\hat{b} = (X^T X)^{-1} X^T y$

$$E(\hat{b}) = E\left[(X^T X)^{-1} X^T y\right] = (X^T X)^{-1} X^T \cdot \underbrace{E(y)}_{Xb}$$

$$= (X^T X)^{-1} X^T X b = b$$

Solution: Estimable Function

solution vectors:

$$b^{(0)} = \begin{bmatrix} \mu^0 \\ \alpha_0^0 \\ \alpha_1^0 \\ \alpha_2^0 \\ \alpha_3^0 \end{bmatrix} ; \quad b^{(1)} = \begin{bmatrix} \mu^1 \\ \alpha_1^1 \\ \alpha_2^1 \\ \alpha_3^1 \end{bmatrix} ; \quad b^{(2)} \dots$$

□ Not looking at individual components of solution vectors  $b^{(j)}$ ; but at linear functions of components:

$$b_1 \cdot \mu^{(j)} + b_2 \cdot \alpha_1^{(j)} + b_3 \cdot \alpha_2^{(j)} + b_4 \cdot \alpha_3^{(j)}$$

$$(1): \alpha_1^{(j)} - \alpha_2^{(j)}$$

$$\Rightarrow 0 \cdot \mu^{(j)} + 1 \cdot \alpha_1^{(j)} + (-1) \cdot \alpha_2^{(j)} + 0 \cdot \alpha_3^{(j)}$$

OHP Picture 4

$l_1 \cdot \mu^{(i)} + l_2 \cdot \alpha_1^{(i)} + l_3 \cdot \alpha_2^{(i)} + l_4 \cdot \alpha_3^{(i)}$

(1) :  $\alpha_1^{(i)} - \alpha_2^{(i)}$   
 $\Rightarrow 0 \cdot \mu^{(i)} + 1 \cdot \alpha_1^{(i)} + (-1) \cdot \alpha_2^{(i)} + 0 \cdot \alpha_3^{(i)}$

$$\underbrace{\begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}}_{q^t} \cdot \underbrace{\begin{bmatrix} \mu^{(i)} \\ \alpha_1^{(i)} \\ \alpha_2^{(i)} \\ \alpha_3^{(i)} \end{bmatrix}}_{b^{(i)}}$$

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(2) :  $\mu^{(i)} + \alpha_1^{(i)} : \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}}_{q^t} \cdot b^{(i)}$

(3) :  $\mu^{(i)} + \frac{1}{2}(\alpha_2^{(i)} + \alpha_3^{(i)}) : \underbrace{\begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}_{q^t}$

General:  
 Linear Function  $q^t b$  is estimable, if  
 $q^t \cdot b^{(i)} = t^T \cdot E(y) = t^T X b^{(i)}$  where  $b^{(i)}$  is a vector

General :

Linear Function  $q^t b$  is estimable, if  $q^t b^{(i)} = t^T E(y) = t^T X b^{(i)}$  where  $b^{(i)}$

why ?

$$q^t = t^T X$$

is a solution to Normal Equations

$$\begin{aligned} q^t \cdot b^{(i)} &= q^t \cdot G X^T y \\ &= t^T X \cdot G X^T y \end{aligned}$$

Because  $X G X^T$  is the same for all choices of  $G$

Definition of  $G$ :

$$X^T X G X^T X = X^T X$$

$$X^T X G X^T X = X^T X$$

$$X G X^T X = X$$

Some other Generalized inverse  $F$  of  $X^T X$ :

$$X^T X F X^T X = X^T X$$

$$X F X^T X = X$$

Generate  $q^T$  for estimable Function:

$$q^T = t^T \cdot X$$

Test whether  $q^T$  leads to estimable Function:

$$q^T \cdot H = q^T$$

Recap:

□ Estimable Function

- Linear function of solutions from solutions of least squares normal equations

- Given solution vector  $b^{(0)} = GX^T y$

where  $G$  is a generalized inverse of  $X^T X$ , i.e.  $X^T X G X^T X = X^T X$

- Def  $q^T \cdot b^{(0)} = t^T \cdot E(y)$