

Recap:

□ Estimable Function

- Linear function of solutions from solutions of least squares normal equations

- Given solution vector  $b^{(0)} = GX^T y$  where  $G$  is a generalized inverse of  $X^T X$ , i.e.  $X^T X G X^T X = X^T X$

- Def  $q^T \cdot b^{(0)} = t^T \cdot E(y)$

because:  $q^T \cdot b^{(0)} = q^T \cdot GX^T y$   
 $= t^T E(y) \cdot GX^T y$   
 $= t^T X b$

$q^T \cdot b^{(0)} = t^T E(y) = t^T X b^{(1)}$   
 $= t^T X G X^T y$ ;  $X G X^T$  is invariant to  $G$



Find  $q^T$  : Determine  $t$  and compute  
 $t^T X = q^T$

Test  $q^T$  is estimable : Compute  $q^T H$  and  
 verify that  $q^T H = q^T$

Contrasts

- Linear combinations of parameters
- In R: contrasts used for a factor can be obtained by function contrasts()

matrix of estimable functions;

		Agus	Limousin	Simmental	
mat_est_fun =	(Intercept)	[ 1	0	0	] → $q^T [2:4]$
	Limousin	[ -1	1	0	
	Simmental	[			

For given solution  $b^{(0)} = \begin{bmatrix} \mu_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$  of the normal equations:  $T_1^0$

matrix of estimable functions;

$$\text{mat\_est\_fun} = \begin{matrix} \text{(Intercept)} & \text{Augus} & \text{Limousin} & \text{Simmental} \\ \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix} \end{matrix}$$

For given solution  $b^{(0)} = \begin{bmatrix} \mu_0^0 \\ \alpha_1^0 \\ \alpha_2^0 \\ \alpha_3^0 \end{bmatrix}$  of the normal equations;  $q^T b^{(0)}$  is estimable

$$q^T = [0 \quad -1 \quad 1 \quad 0]$$

$$\Rightarrow \text{Limousin-Effekt: } q^T \cdot b^{(0)} = 0 \cdot \mu_0^0 + (-1) \alpha_1^0 + (1) \alpha_2^0 + 0 \alpha_3^0$$

By default in R: treatment contrast  
 - First factor level: control.  
 other levels are taken as treatments.