

Recap : 2023-05-15

□ Mixed models because

$\text{cov}(y_4, y_5) = 0$ because animals 4 and 5 are unrelated

$\text{cov}(y_4, y_6) \neq 0$ because 4 and 6 are half-sibs

□ To be able to integrate a given covariance structure due to ancestral relationship in the data, we have to use mixed linear effect models, where the effect responsible for the covariance structure is a random effect with suitable variance-covariance matrix (D)

□ Model : $y = Xb + Zu + e$

Annotations:
- b : fixed
- u : random effects
- e : random residuals

OHP Picture 2

□ First example: Sire Model

$$y = Xb + Zs + e$$

vector random sire effects

□ Example data: 3 Sires: 1, 3, 4

⇒ s has 3 elements $s^T = [s_1, s_3, s_4]$

□ Put information from data into model:

$$y = \begin{bmatrix} 4.5 \\ 2.9 \\ 3.9 \\ 3.5 \\ 5.0 \end{bmatrix} \rightarrow M; \quad b = \begin{bmatrix} b_n \\ b_f \end{bmatrix}; \quad s = \begin{bmatrix} s_1 \\ s_3 \\ s_4 \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

X and Z are design matrices

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ b_n & b_f \end{bmatrix} \quad Z = \begin{matrix} & \begin{matrix} s_1 & s_3 & s_4 \end{matrix} \\ \begin{matrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

OHP Picture 3

□ First example: Sire Model

$$y = Xb + Zs + e$$

vector random sire effects

□ Example data: 3 Sires: 1, 3, 4

⇒ S has 3 elements $S^T = [s_1, s_3, s_4]$

□ Put information from data into model:

$$y = \begin{bmatrix} 4.5 \\ 2.9 \\ 3.9 \\ 3.5 \\ 5.0 \end{bmatrix} \rightarrow M; \quad b = \begin{bmatrix} b_n \\ b_f \end{bmatrix}; \quad s = \begin{bmatrix} s_1 \\ s_3 \\ s_4 \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_5 \end{bmatrix}$$

X and Z are design matrices

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ b_n & b_f \end{bmatrix}$$

$$Z = \begin{matrix} & \begin{matrix} s_1 & s_3 & s_4 \end{matrix} \\ \begin{matrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

OHP Picture 4

$$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + D^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$$

$$R^{-1} = I \sigma_e^{-2}$$

$$\begin{bmatrix} X^T I \sigma_e^{-2} X & X^T I \sigma_e^{-2} Z \\ Z^T I \sigma_e^{-2} X & Z^T I \sigma_e^{-2} Z + D^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T I \sigma_e^{-2} y \\ Z^T I \sigma_e^{-2} y \end{bmatrix}$$

↗ number
↙ Identity

⇒ Ignore multiplication with I

⇒ Factor out σ_e^{-2} by Multiplication of both sides with σ_e^2

$$\Rightarrow \begin{bmatrix} X^T X & X^T Z \\ Z^T X & Z^T Z + A_S^{-1} \sigma_s^2 \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{s} \end{bmatrix} = \begin{bmatrix} X^T y \\ Z^T y \end{bmatrix}$$

MSE
↗ σ_s^2
↙ σ_e^2
↖ σ_e^2
↘ σ_s^2

$$D = \text{var}(s) = \begin{bmatrix} \text{var}(s_1) & \text{cov}(s_1, s_2) & \dots \end{bmatrix}$$

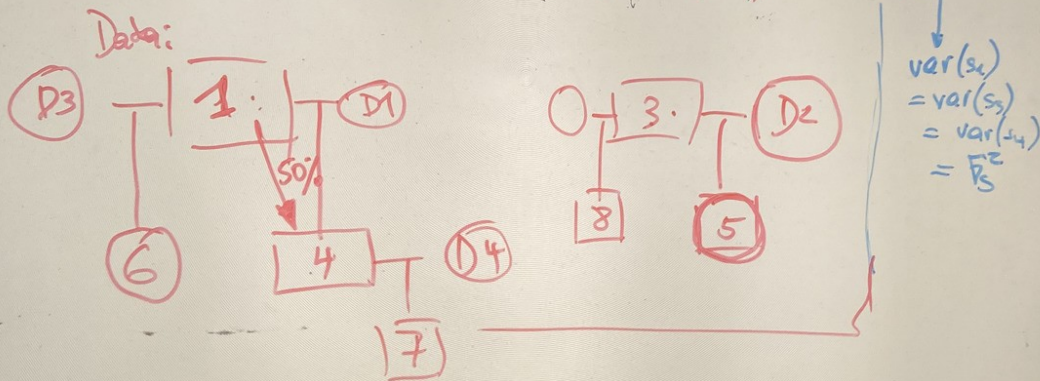
$$= A_S \cdot \sigma_s^2 \rightarrow D^{-1} = A_S^{-1} \cdot \sigma_s^{-2}$$

OHP Picture 5

A_S : Sire relationship matrix

$D = \text{var}(S)$; Example $S = \begin{bmatrix} S_1 \\ S_2 \\ S_4 \end{bmatrix}$

$$D = \text{var}(S) = \begin{bmatrix} \text{var}(S_1) & \text{cov}(S_1, S_2) & \text{cov}(S_1, S_4) \\ \text{cov}(S_2, S_1) & \text{var}(S_2) & \text{cov}(S_2, S_4) \\ \text{cov}(S_4, S_1) & \text{cov}(S_4, S_2) & \text{var}(S_4) \end{bmatrix}$$



$$D = \begin{bmatrix} \sigma^2 & 0 & 1/2 \cdot \sigma^2 \\ 0 & \sigma^2 & 0 \\ 1/2 \cdot \sigma^2 & 0 & \sigma^2 \end{bmatrix} = A_S \cdot \sigma^2$$

Animal Model.

□ Predicted breeding values for all animals

□ Model : $y = Xb + Zu + e$ $\rightarrow u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_8 \end{bmatrix}$

□ $D = \text{var}(u) = \begin{bmatrix} \text{var}(u_1) & \text{cov}(u_1, u_2) & \dots \\ \text{cov}(u_2, u_1) & \text{var}(u_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$

$D = A \cdot \sigma_u^2$

genetic - additive variance,
of ten specified $h^2 = \sigma_u^2 / \sigma_p^2$

$\sigma_p^2 = \sigma_u^2 + \sigma_e^2$

$\Rightarrow \lambda = \sigma_e^2 / \sigma_u^2$

OHP Picture 7

