

# Interactions

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# Definition

- ▶ Effect of given predictor variable depends on level or value of other predictor variable
- ▶ Examples:
  - ▶ Regression of Body Weight on Breast Circumference is different for different breeds
  - ▶ Effect of Breed on Body Weight is different for different male and female animals

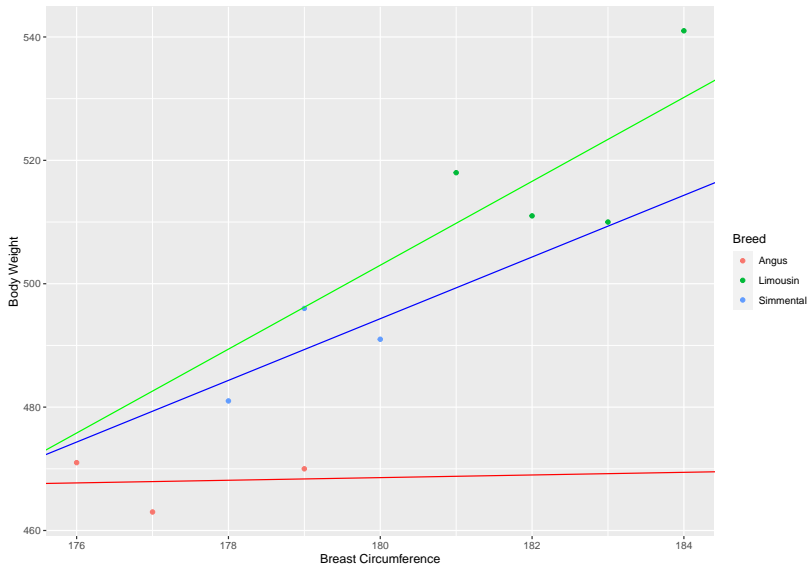
# Types of Interactions

- ▶ continuous by continuous
- ▶ continuous by categorical
- ▶ categorical by categorical

## Continuous by Categorical

- ▶ In a model, expected value of response depends on regression variable plus discrete factor
- ▶ Example: Regression of Body Weight on Breast Circumference plus the factor Breed
- ▶ Interaction is present, if regression of Body Weight on Breast Circumference is different for different breeds

# Interaction Plot



## Interaction Model

- ▶ Start with model without interactions

$$y_i = b_0 + b_1 \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

- ▶ Assume linear relationship of  $b_1$  with Breed

$$b_1 = a + b_4 \times BrLi + b_5 \times BrSi$$

- ▶ Insert

$$y_i = b_0 + (a + b_4 \times BrLi + b_5 \times BrSi) \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

- ▶ Simplify

$$y_i = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i \\ + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i + e_i$$

## Continuous by Continuous

- ▶ Similar to continuous by categorical
- ▶ No interaction

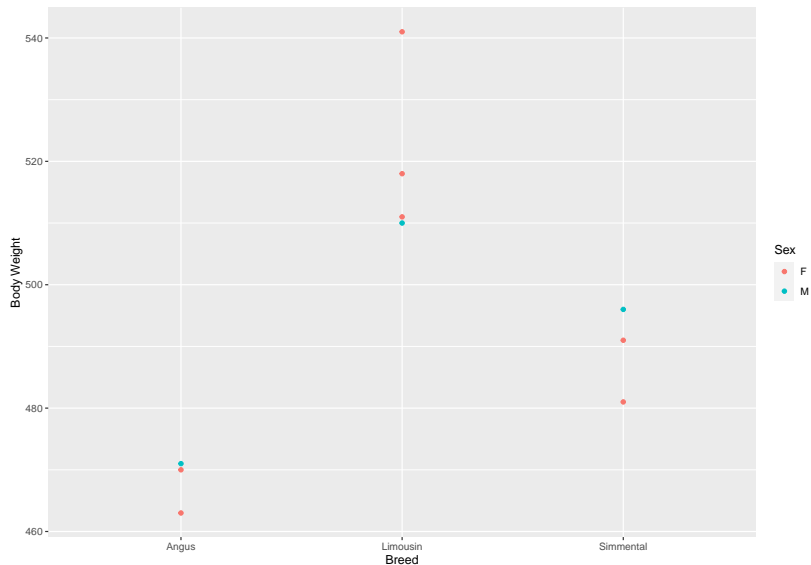
$$y_0 = b_0 + b_1 \times BC_i + b_2 \times HE_i + e_i$$

- ▶ Interaction by dependence of one regression coefficient on other coefficient

$$b_1 = b_3 + b_4 \times HE_i$$

$$y_0 = b_0 + b_2 \times HE_i + b_3 \times BC_i + b_4 \times HE_i \times BC_i + e_i$$

# Categorical by Categorical





# Model Matrix

```
## (Intercept) BreedLimousin BreedSimmental SexM BreedLimousin:SexM
## 1 1 0 0 1 0
## 2 1 0 0 0 0
## 3 1 0 0 0 0
## 4 1 1 0 0 0
## 5 1 1 0 0 0
## 6 1 1 0 1 1
## 7 1 1 0 0 0
## 8 1 0 1 0 0
## 9 1 0 1 1 0
## 10 1 0 1 0 0
## BreedSimmental:SexM
## 1 0
## 2 0
## 3 0
## 4 0
## 5 0
## 6 0
## 7 0
## 8 0
## 9 1
## 10 0
## attr("assign")
## [1] 0 1 1 2 3 3
## attr("contrasts")
## attr("contrasts")$Breed
## [1] "contr.treatment"
##
## attr("contrasts")$Sex
## [1] "contr.treatment"
```

# Summary

```
##
## Call:
## lm(formula = 'Body Weight' ~ Breed * Sex, data = tbl_flem_bw_br_sex)
##
## Residuals:
##      1      2      3      4      5      6      7
## 3.726e-15 -3.500e+00  3.500e+00 -5.333e+00 -1.233e+01 -1.703e-15  1.767e+01
##      8      9     10
## -5.000e+00 -6.458e-16  5.000e+00
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      466.50      8.42    55.404 6.35e-07 ***
## BreedLimousin     56.83     10.87     5.228 0.00639 **
## BreedSimmental    19.50     11.91     1.638 0.17685
## SexM               4.50     14.58     0.309 0.77306
## BreedLimousin:SexM -17.83     20.04    -0.890 0.42389
## BreedSimmental:SexM  5.50     20.62     0.267 0.80291
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.91 on 4 degrees of freedom
## Multiple R-squared:  0.8981, Adjusted R-squared:  0.7706
## F-statistic: 7.048 on 5 and 4 DF, p-value: 0.04092
```