Interactions

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Recap: Contrasts
> different contrasts,
> treatment per default
> custom contrast

Interactions

> genetic evaluation

> correction for environmental effect

> examples of interactions Herd*Year*Season

Definition

Interactions occur only, if there is more than one predictor in the model

- Effect of given predictor variable depends on level or value of other predictor variable
- Examples:
  - Regression of `Body Weight` on `Breast Circumference` is different for different breeds
  - Effect of `Breed` on `Body Weight` is different for different male and female animals

interaction between breed and sex

if the regression coefficient of body weight on breast circumference is different for different breeds, then there is an interaction between breast circumference and breed.

Types of Interactions

- continuous by continuous
- continuous by categorical
- categorical by categorical

interaction breast circumference and height

breast circumference and breed

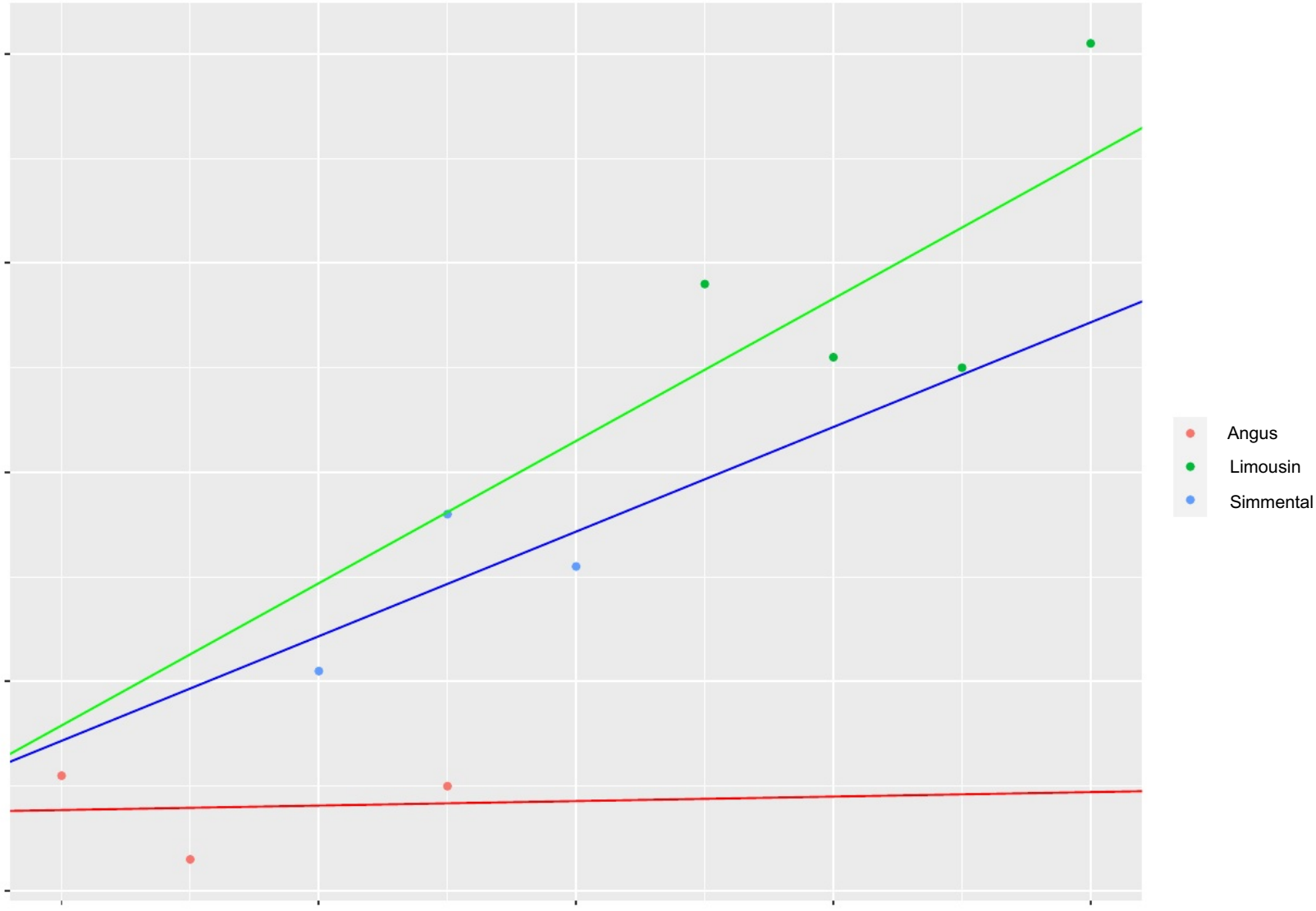
breed and sex

Continuous by Categorical

pre-requisite: model must contain a regression variable (breast circumference) plus a categorical factor (like breed)

- In a model, expected value of response depends on regression variable plus discrete factor
- Example: Regression of `Body Weight` on `Breast Circumference` plus the factor `Breed`
- Interaction is present, if regression of `Body Weight` on `Breast Circumference` is different for different breeds

Interaction Plot



Interaction Model

- Start with model without interactions

intercept

Regressioncoefficient for Breast Circumference

$$y_i = b_0 + b_1 \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

breed effects using treatment contrasts

- Assume linear relationship of b_1 with `Breed`

$$b_1 = a + b_4 \times BrLi + b_5 \times BrSi$$

- Insert

$$y_i = b_0 + (a + b_4 \times BrLi + b_5 \times BrSi) \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + e_i$$

- Simplify

regression of Bw on BC

breed effects

intercept

$$y_i = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i + e_i$$

terms depending on product of BC and breed effects

Meaning of interaction terms:
> Used the regression coefficient to quantify the effect that an increment in predictor variable has on the response
==> What is the effect of an increase of 1 cm in BC on the expected Body Weight?
==> Show example

Expected increase in body weight, if breast circumference increases by 1 cm

```{r}
lm_bw_bc <- lm(`Body Weight` ~ `Breast Circumference`, data = tbl_bw_inter_bc_br)
summary(lm_bw_bc)
```

```
Call:
lm(formula = `Body Weight` ~ `Breast Circumference`, data = tbl_bw_inter_bc_br)

Residuals:
     Min       1Q   Median       3Q      Max
-17.3941  -6.5525  -0.0673   9.3707  13.2594

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)          -1065.115    255.483  -4.169 0.003126 **
`Breast Circumference`     8.673      1.420   6.108 0.000287 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 11.08 on 8 degrees of freedom
Multiple R-squared:  0.8234,    Adjusted R-squared:  0.8014
F-statistic: 37.31 on 1 and 8 DF,  p-value: 0.000287
```

With interactions:
> 1. Angus animal: Increase breast circumference by 1cm ==> Expected increase in body weight?

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$$E(y_i) = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i$$

For two Angus animals which have a difference in breast circumference of 1 cm: , the expected difference in body weight will be a kg.

Animal 1: 174 cm
Animal 2: 175 cm

Expected difference in body weight between animal 2 and 1: a kg

For Limousin:

$$E(y_i) = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i$$

inserting:

$$E(y_i) = b_0 + a \times BC_i + b_2 \times 1 + b_3 \times 0 + b_4 \times 1 \times BC_i + b_5 \times 0 \times BC_i$$

Increament for 1 cm for Li:

$$= a + b_4$$

In R:

```{r}
lm_bw_bc_br_inter <- lm(`Body Weight` ~ `Breast Circumference` + Breed + `Breast Circumference`:Breed, data = tbl_bw_inter_bc_br)
summary(lm_bw_bc_br_inter)
```

```
Call:
lm(formula = `Body Weight` ~ `Breast Circumference` + Breed +
    `Breast Circumference`:Breed, data = tbl_bw_inter_bc_br)

Residuals:
      1       2       3       4       5       6       7       8       9      10
  3.286  -4.929   1.643   8.200  -5.600 -13.400  10.800  -3.333   6.667  -3.333

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                       430.0000   917.1235   0.469    0.664
`Breast Circumference`              0.2143     5.1716   0.041    0.969
BreedLimousin                   -1151.0000  1293.2741  -0.890    0.424
BreedSimmental                   -835.6667  1685.4451  -0.496    0.646
`Breast Circumference`:BreedLimousin    6.5857     7.1908   0.916    0.412
`Breast Circumference`:BreedSimmental   4.7857     9.4420   0.507    0.639

Residual standard error: 11.17 on 4 degrees of freedom
Multiple R-squared:  0.9103,    Adjusted R-squared:  0.7981
F-statistic: 8.115 on 5 and 4 DF,  p-value: 0.03212
```

$$E(y_i) = b_0 + a \times BC_i + b_2 \times BrLi_i + b_3 \times BrSi_i + b_4 \times BrLi \times BC_i + b_5 \times BrSi \times BC_i$$

Continuous by Continuous

Influence of breast circumference and height on body weight

- Similar to continuous by categorical
- No interaction

$$y_0 = b_0 + b_1 \times BC_i + b_2 \times HE_i + e_i$$

- Interaction by dependence of one regression coefficient on other coefficient

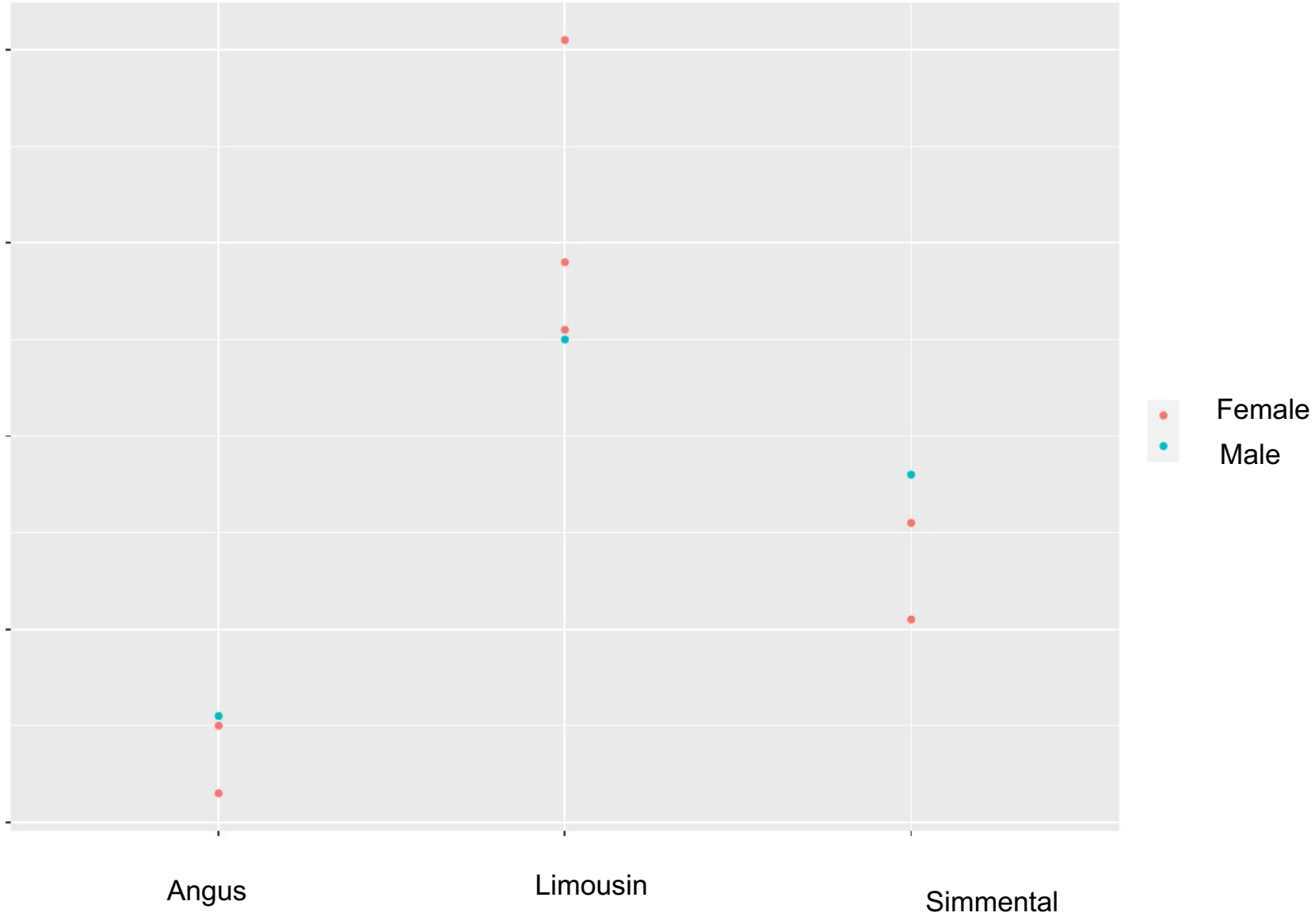
$$b_1 = b_3 + b_4 \times HE_i$$

Interaction depends on product of predictor variables

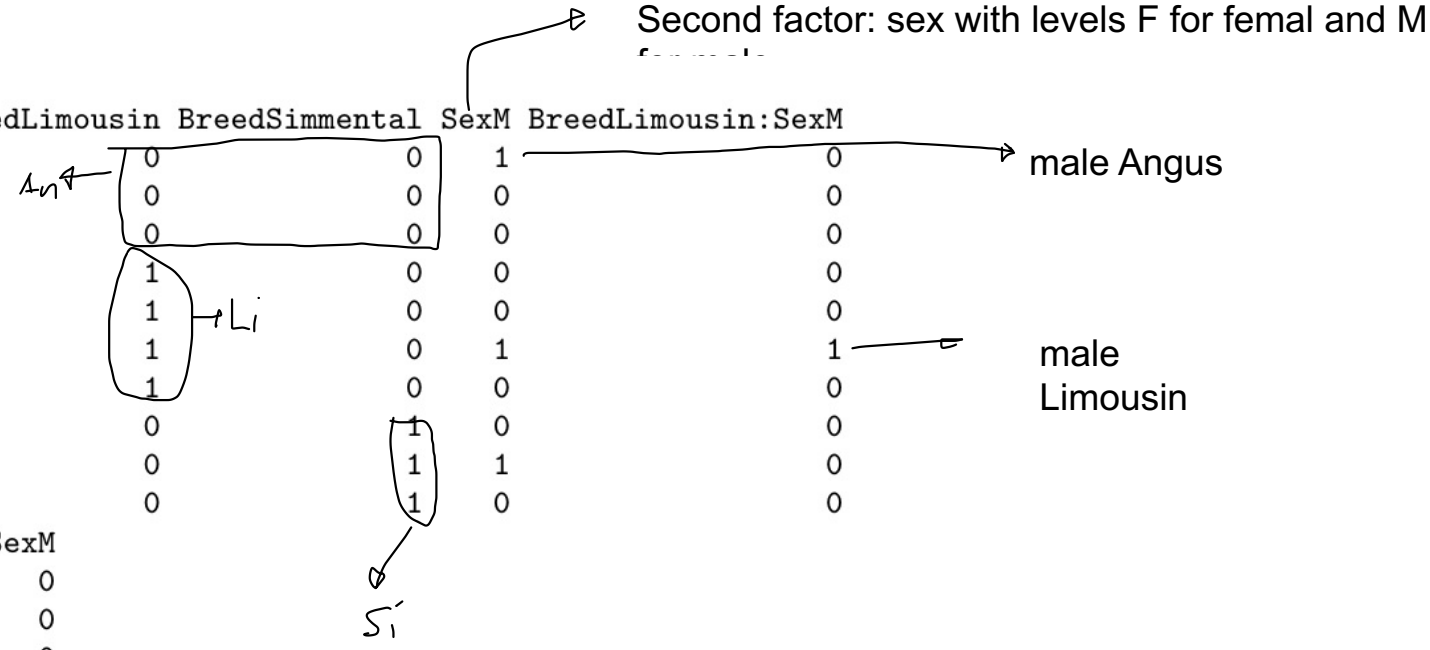
$$y_0 = b_0 + b_2 \times HE_i + b_3 \times BC_i + b_4 \times HE_i \times BC_i + e_i$$

Categorical by Categorical

Different influence of breed on body weight depending second factor such as sex



Model Matrix




```
##    (Intercept) BreedLimousin BreedSimmental SexM BreedLimousin:SexM
## 1            1             0               0    1                  0
## 2            1             0               0    0                  0
## 3            1             0               0    0                  0
## 4            1             1               0    0                  0
## 5            1             1               0    0                  0
## 6            1             1               0    1                  1
## 7            1             1               0    0                  0
## 8            1             0               1    0                  0
## 9            1             0               1    1                  0
## 10           1             0               1    0                  0
##    BreedSimmental:SexM
## 1                    0
## 2                    0
## 3                    0
## 4                    0
## 5                    0
## 6                    0
## 7                    0
## 8                    0
## 9                    1
## 10                   0
## attr(,"assign")
## [1] 0 1 1 2 3 3
## attr(,"contrasts")
## attr(,"contrasts")$Breed
## [1] "contr.treatment"
##
## attr(,"contrasts")$Sex
## [1] "contr.treatment"
```

Summary

```
##
## Call:
## lm(formula = `Body Weight` ~ Breed * Sex, data = tbl_flem_bw_br_sex)
##
## Residuals:
##          1          2          3          4          5          6          7
##  3.726e-15 -3.500e+00  3.500e+00 -5.333e+00 -1.233e+01 -1.703e-15  1.767e+01
##          8          9         10
## -5.000e+00 -6.458e-16  5.000e+00
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)           466.50       8.42  55.404 6.35e-07 ***
## BreedLimousin          56.83      10.87   5.228  0.00639 **
## BreedSimmental         19.50      11.91   1.638  0.17685
## SexM                    4.50      14.58   0.309  0.77306
## BreedLimousin:SexM    -17.83      20.04  -0.890  0.42389
## BreedSimmental:SexM     5.50      20.62   0.267  0.80291
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.91 on 4 degrees of freedom
## Multiple R-squared:  0.8981, Adjusted R-squared:  0.7706
## F-statistic: 7.048 on 5 and 4 DF,  p-value: 0.04092
```