Overhead Lecture 4

Peter von Rohr

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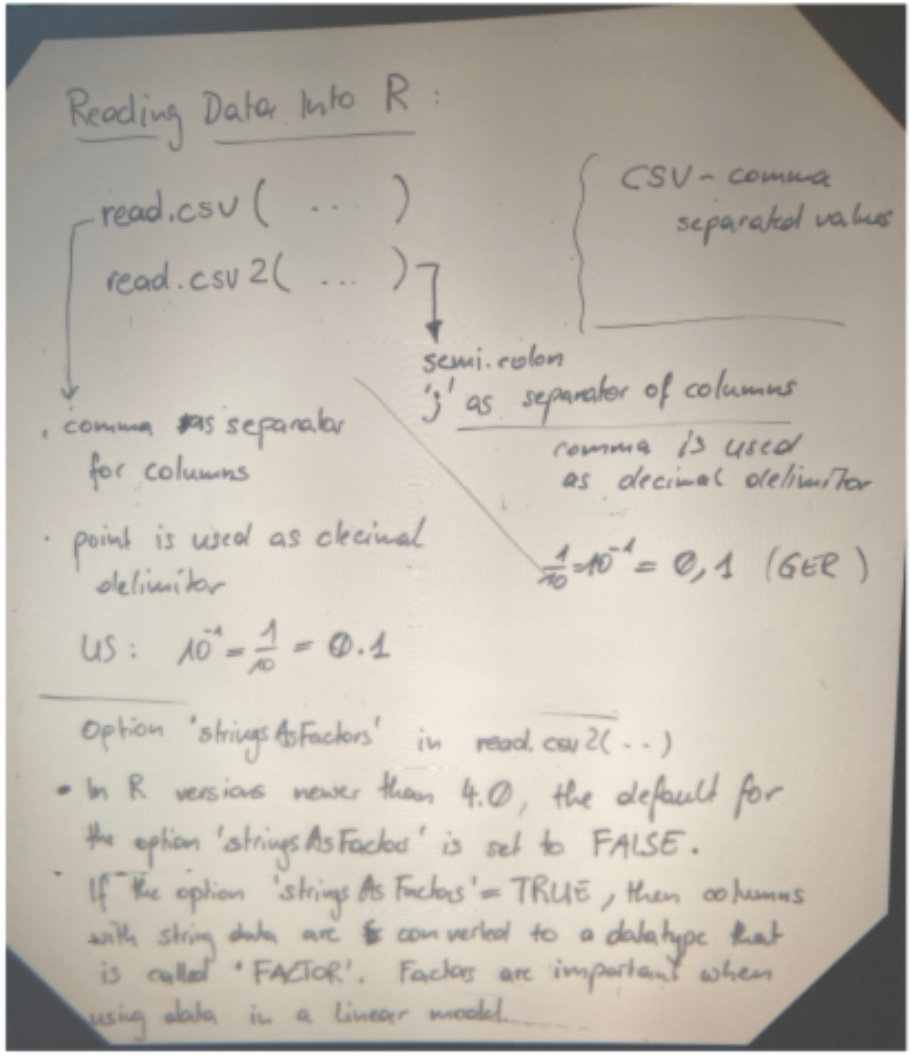
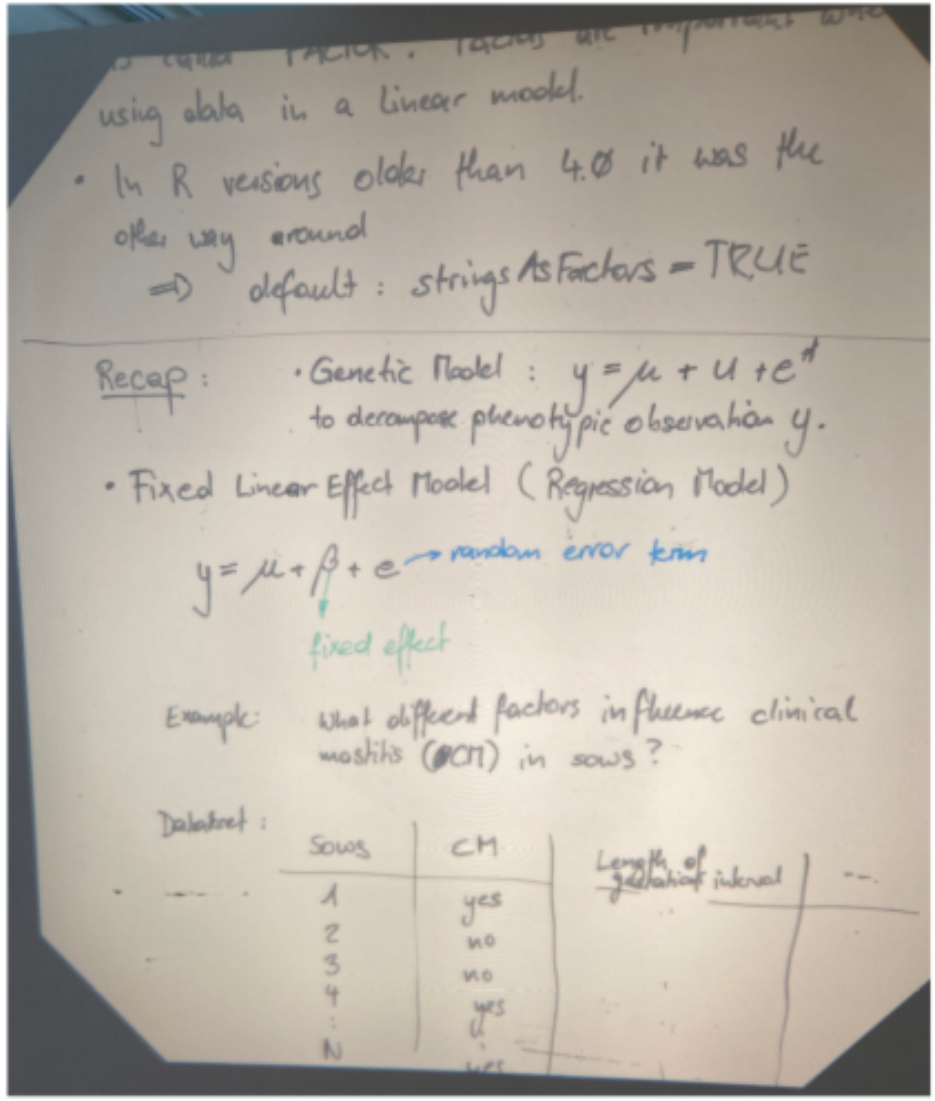
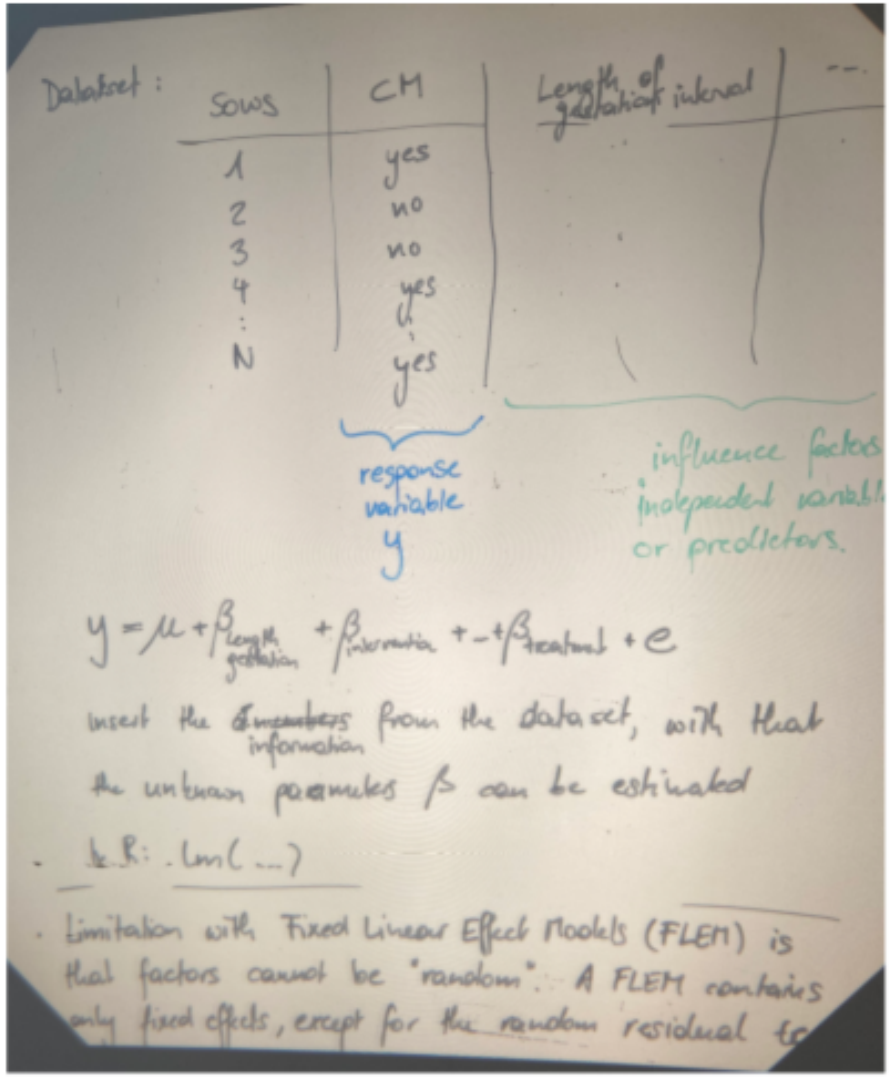
Import Data in R



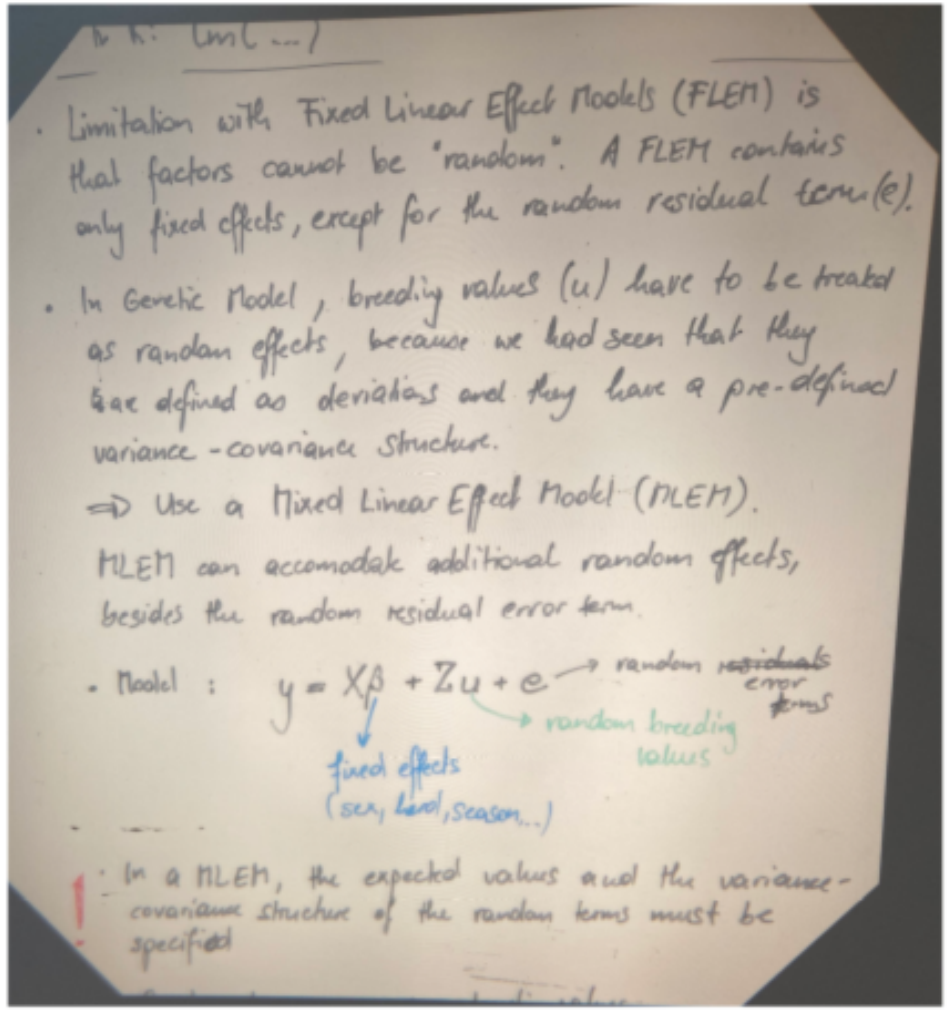
Recap Genetic Model



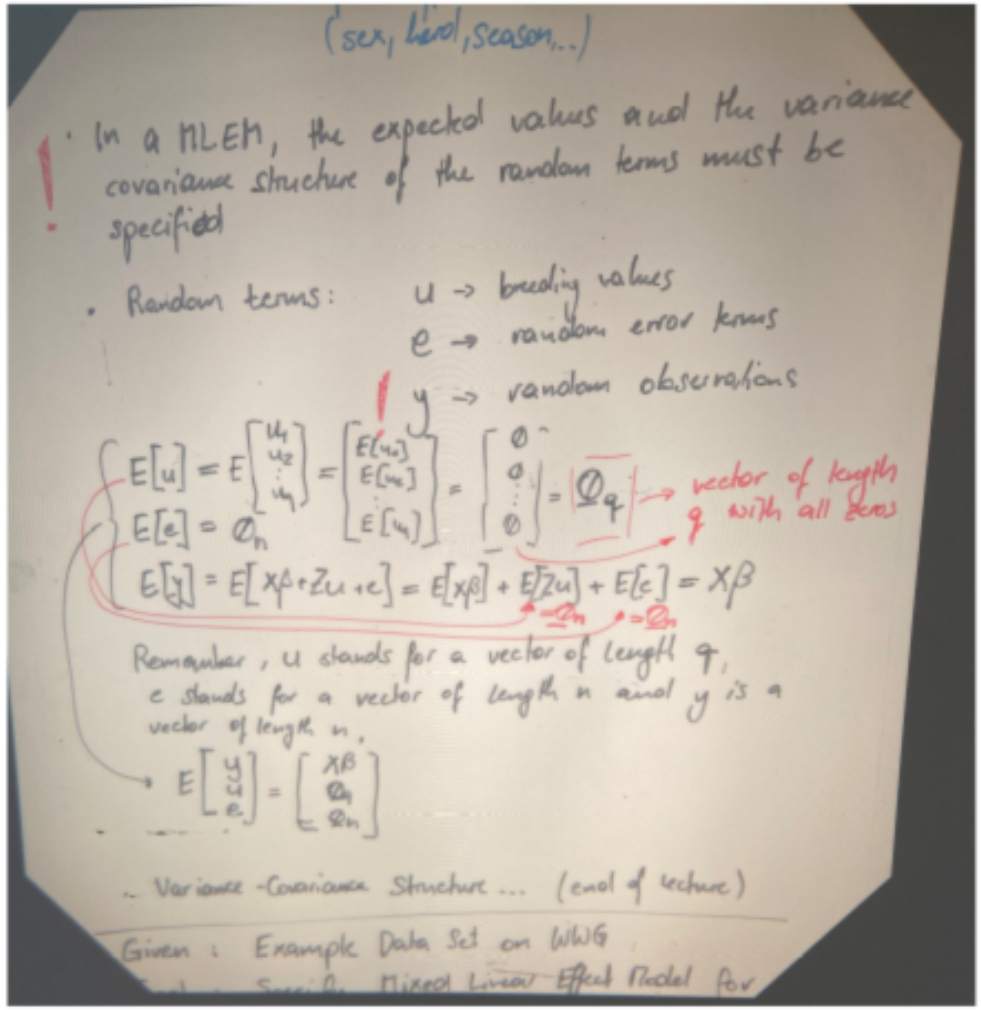
Fixed Linear Effects Model



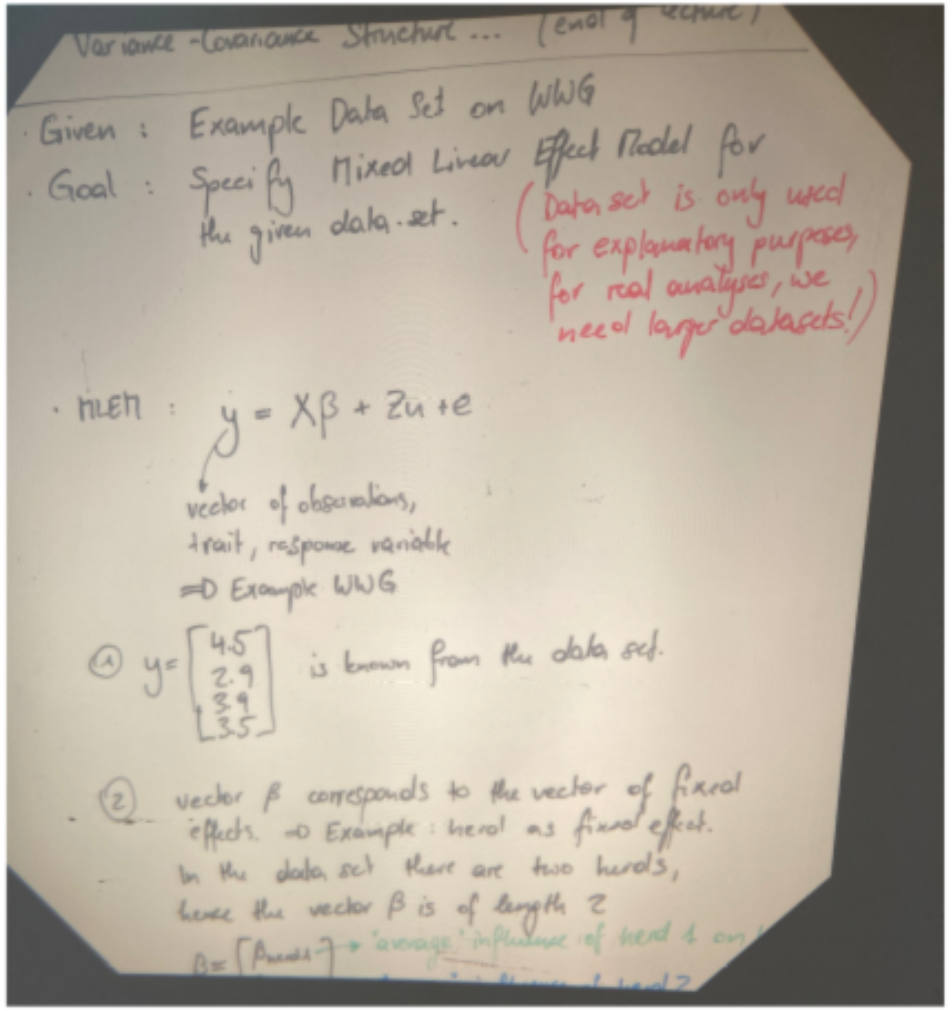
Mixed Linear Effects Model



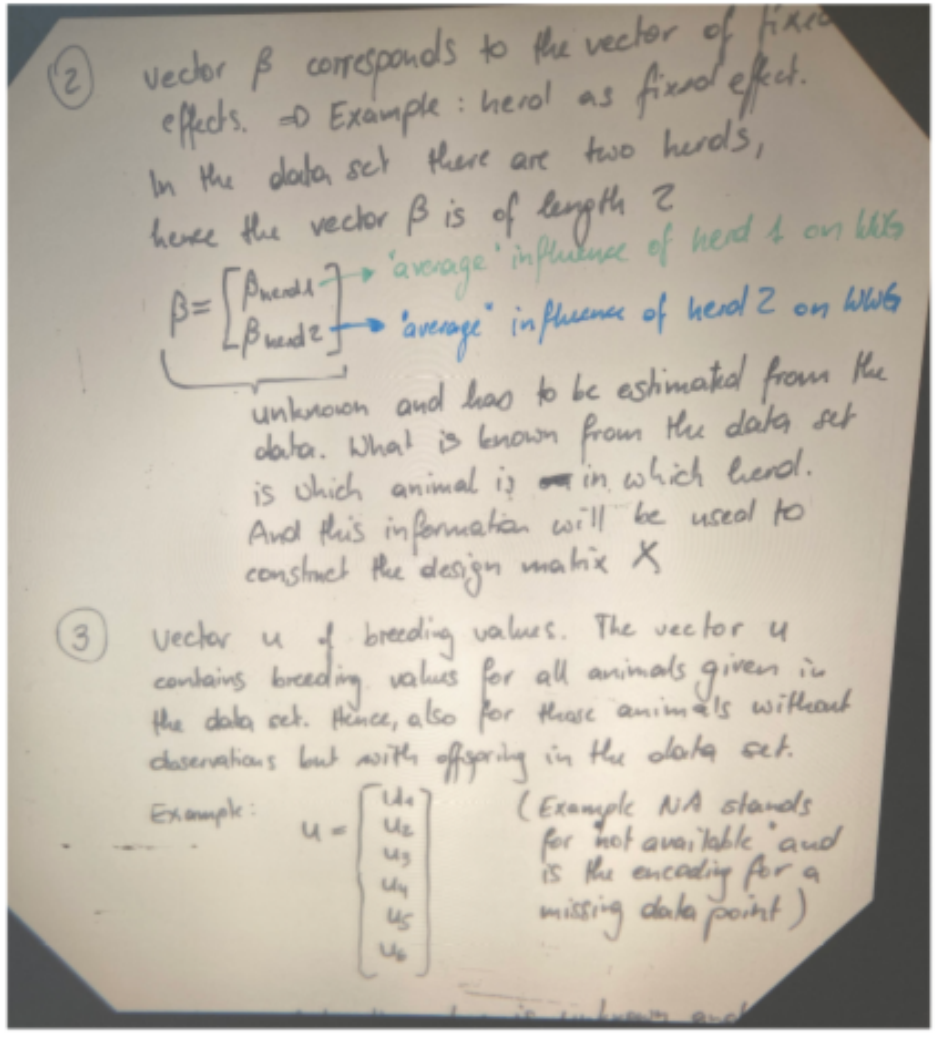
Model Specification



Model Components



Fixed and Random Effects



(2) Vector β corresponds to the vector of fixed effects. ⇒ Example: herd as fixed effect. In the data set there are two herds, hence the vector β is of length 2

$$\beta = \begin{bmatrix} \beta_{herd1} \\ \beta_{herd2} \end{bmatrix}$$

β_{herd1} → "average" influence of herd 1 on WWG

β_{herd2} → "average" influence of herd 2 on WWG

unknown and has to be estimated from the data. What is known from the data set is which animal is in which herd. And this information will be used to construct the design matrix X

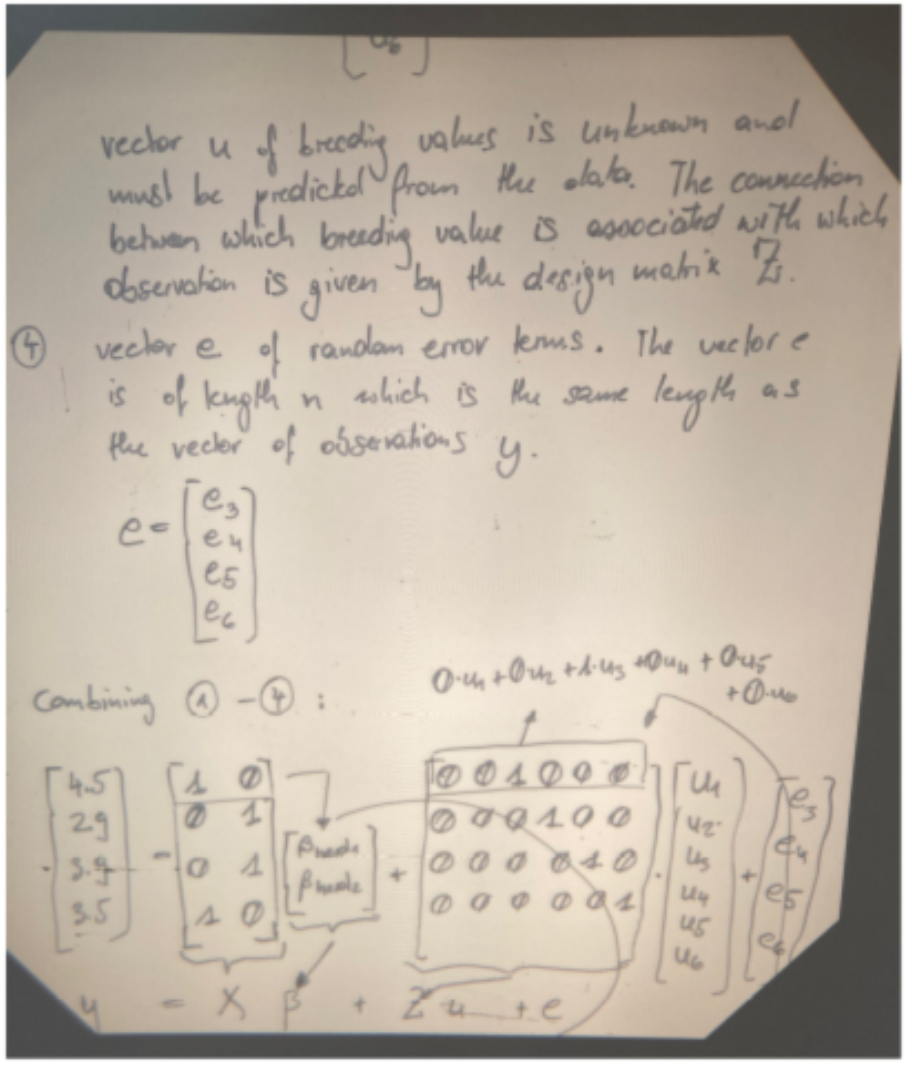
(3) Vector u of breeding values. The vector u contains breeding values for all animals given in the data set. Hence, also for those animals without observations but with offspring in the data set.

Example:

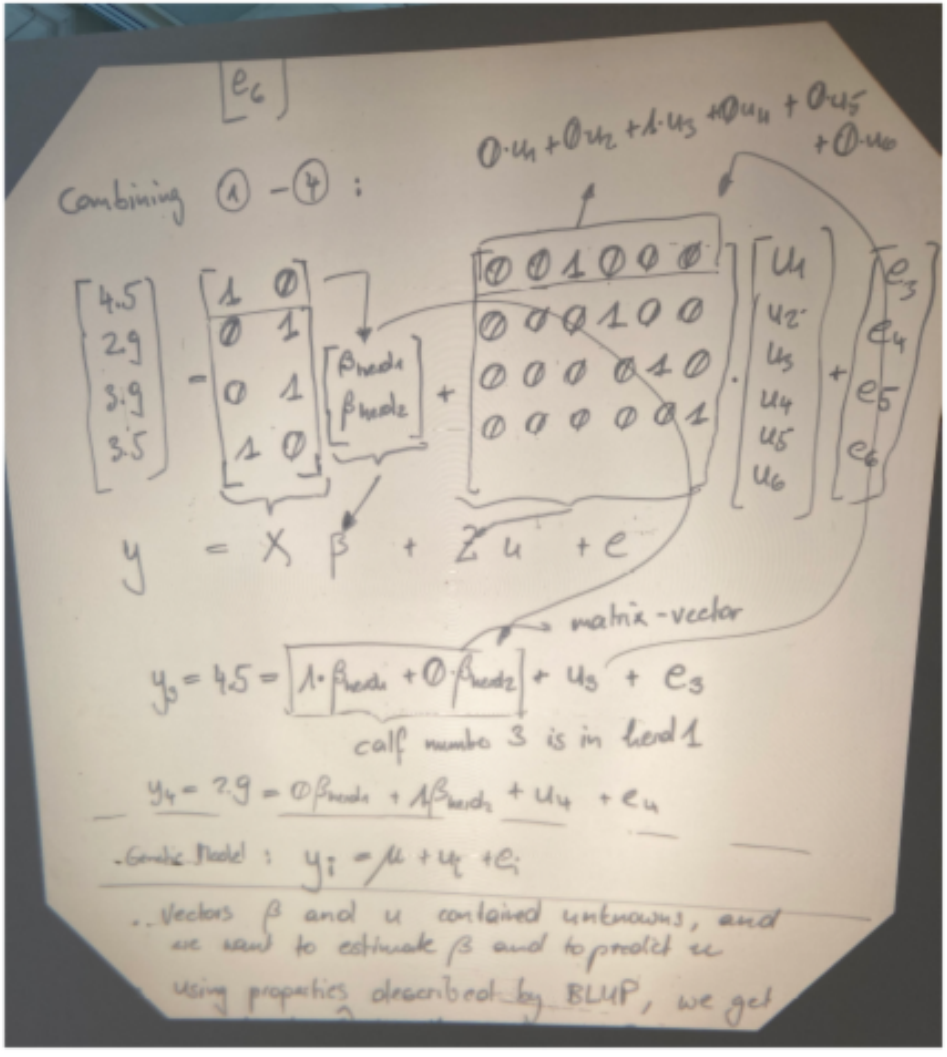
$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

(Example NA stands for "not available" and is the encoding for a missing data point)

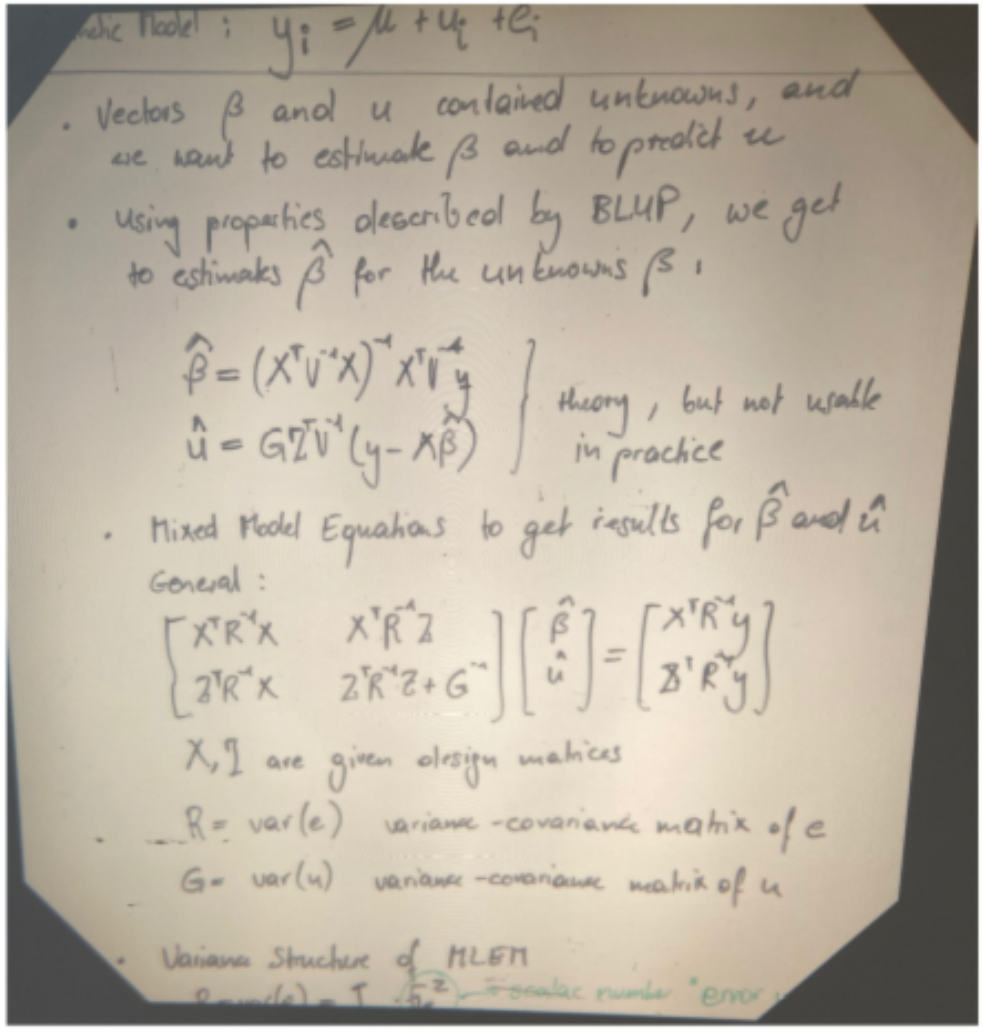
Combining Components



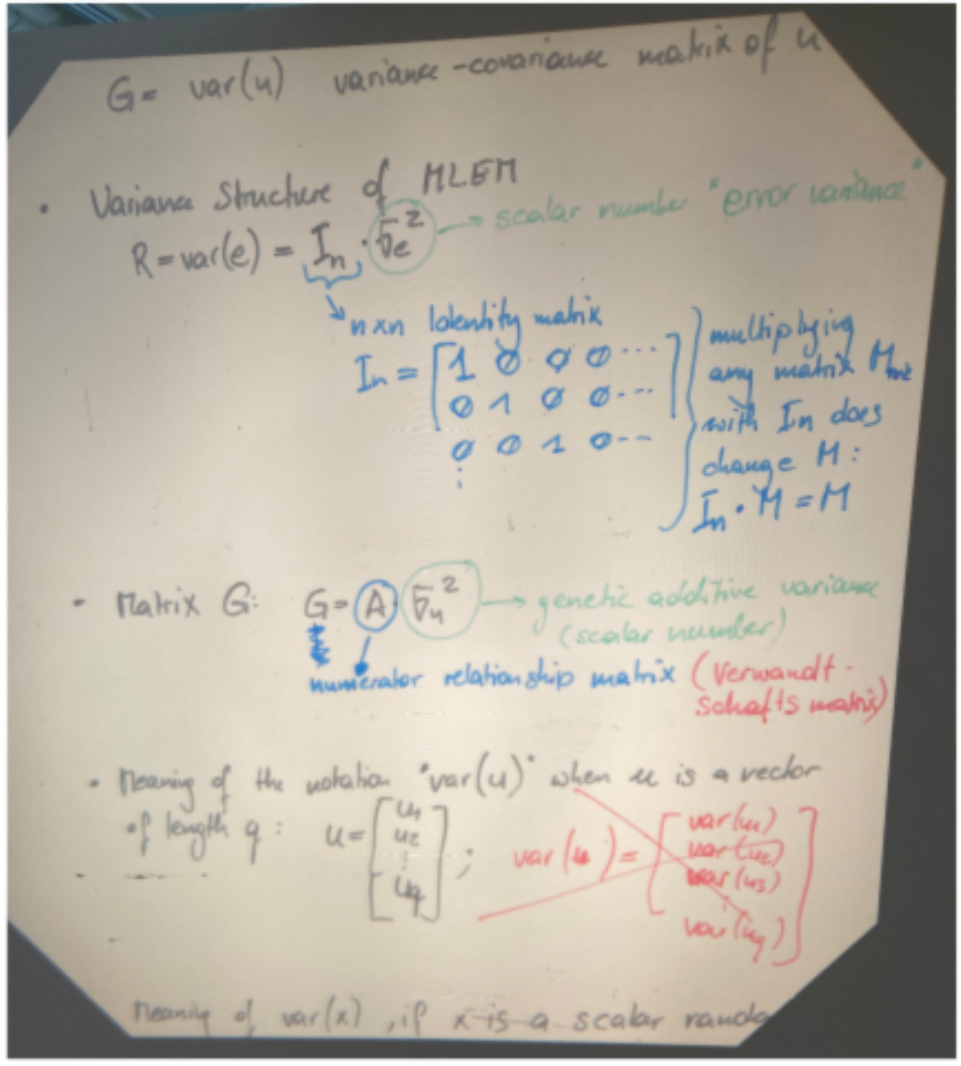
Numerical Example



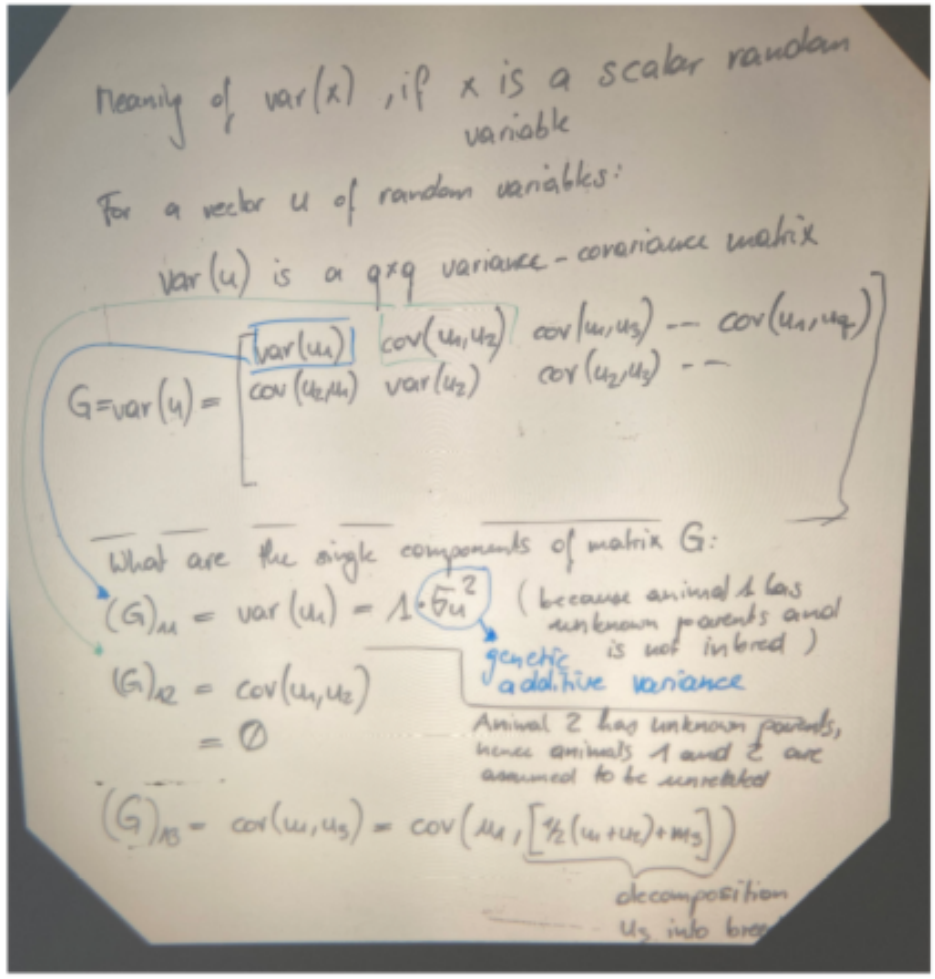
Solutions


Model: $y_i = \mu + u_i + e_i$
• Vectors β and u contained unknowns, and we want to estimate β and to predict u
• Using properties described by BLUP, we get to estimate $\hat{\beta}$ for the unknowns β:
$\hat{\beta} = (X^T V^{-1} X)^{-1} X^T V^{-1} y$
$\hat{u} = G Z^T V^{-1} (y - X\hat{\beta})$
theory, but not usable in practice
• Mixed Model Equations to get results for $\hat{\beta}$ and $\hat{u}$
General:
$\begin{bmatrix} X^T R^{-1} X & X^T R^{-1} Z \\ Z^T R^{-1} X & Z^T R^{-1} Z + G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^T R^{-1} y \\ Z^T R^{-1} y \end{bmatrix}$
X, Z are given design matrices
• $R = \text{var}(e)$ variance-covariance matrix of e
$G = \text{var}(u)$ variance-covariance matrix of u
• Variance Structure of MLEM

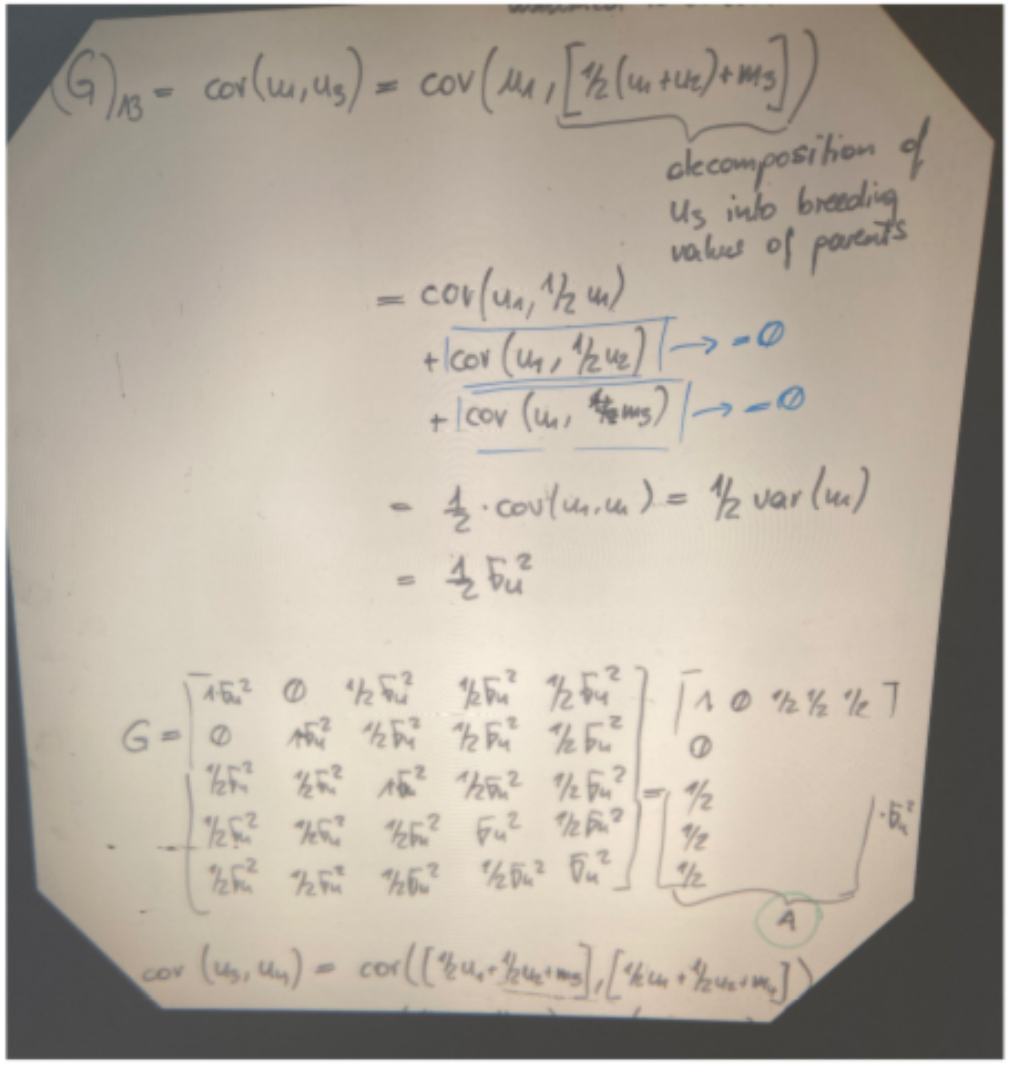
Variance Structure



Genetic Variance



Numerator Relationship Matrix



NRM II