Single Step GBLUP

is based on a breeding-value based model.
In single step, there is no reference population, but the complete population is split into four groups

- Use a mixed linear effect model
- Genomic breeding values g are random effects

$$y = Xb + Zg + e$$

with

- $E(e) = 0, \; var(e) = I * \sigma_e^2$
- $E(g) = 0, \; var(g) = G * \sigma_g^2$
- Genomic relationship matrix G

	I with obs.	I without obs
with SNP	I old male	I young animals
without SNP	I old female	I production

Solution Via Mixed Model Equations

- All animals have genotypes and observations

$$\begin{bmatrix} X^TX & X^TZ \\ Z^TX & Z^TZ + \lambda * G^{-1} \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{g} \end{bmatrix} = \begin{bmatrix} X^Ty \\ Z^Ty \end{bmatrix}$$

with $\lambda = \sigma_e^2/\sigma_g^2$.

Animals Without Observations

- Young animals do not have observations
- Partition $\hat{g}$ into
  - $\hat{g}_1$ animals with observations and
  - $\hat{g}_2$ animals without observations
- Resulting Mixed Model Equations are (assume $\lambda = 1$)

$$\left[\begin{array}{lll} X^TX & X^TZ & 0 \\ Z^TX & Z^TZ + G^{(11)} & G^{(12)} \\ 0 & G^{(21)} & G^{(22)} \end{array}\right] \left[\begin{array}{c} \hat{b} \\ \hat{g}_1 \\ \hat{g}_2 \end{array}\right] = \left[\begin{array}{c} X^Ty \\ Z^Ty \\ 0 \end{array}\right]$$

Predicted Genomic Breeding Values

- Last line of Mixed model equations

$$G^{(21)} \cdot \hat{g}_1 + G^{(22)} \cdot \hat{g}_2 = 0$$

Solutions

- Solving for $\hat{g}_2$

$$\hat{g}_2 = -(G^{(22)})^{-1} \cdot G^{(21)} \cdot \hat{g}_1$$