Livestock Breeding and Genomics - Solution 1

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Problem 1: Vectors

Given are the following two vectors v and w. Compute

- the sum $v + w$,
- the difference $v - w$ and
- the dot product $v \cdot w$.

$$v = \left[\begin{array}{c} 3 \\ -5 \\ 1 \\ 9 \end{array}\right], \quad w = \left[\begin{array}{c} 1 \\ 9 \\ -12 \\ 27 \end{array}\right]$$

Solution

The sum $v + w = \left[\begin{array}{c} 3 + 1 \\ -5 + 9 \\ 1 + (-12) \\ 9 + 27 \end{array}\right] = \left[\begin{array}{c} 4 \\ 4 \\ -11 \\ 36 \end{array}\right]$,

the difference $v - w = \left[\begin{array}{c} 3 - 1 \\ -5 - 9 \\ 1 - (-12) \\ 9 - 27 \end{array}\right] = \left[\begin{array}{c} 2 \\ -14 \\ 13 \\ -18 \end{array}\right]$

The dot product $v \cdot w = 3 * 1 + (-5) * 9 + 1 * (-12) + 9 * (-27) = 189$

Problem 2: Vectors in R

Verify your computations from Problem 1 in R. Start by assigning the vectors v and w with the components given in Problem 1. If you are not sure how to assign vectors in R, please have a look at https://bookdown.org/rdpeng/rprogdatascience/r-nuts-and-bolts.html#creating-vectors and its the corresponding video at https://youtu.be/w8_XdYI3reU

Solution

```r
v <- c(3, -5, 1, 9)
w <- c(1, 9, -12, 27)
```

Now we do the computations.

The sum $v + w$ is

```
v+w
```

```
## [1]   4   4 -11  36
```

The difference $v - w$ is

```
v-w
```

```
## [1]   2 -14  13 -18
```

and the dot-product is

```
crossprod(v,w)
```

```
##      [,1]
## [1,]  189
```

or

```
v %*% w
```

```
##      [,1]
## [1,]  189
```

Please note: Although the R-function is called `crossprod()` what is computed is the dot product between the two vectors. The function name `crossprod()` is used because in Statistics the product (X^TX) of a transposed matrix (X^T) and itself (X) is called a matrix crossproduct. This has nothing to do with the crossproduct $v \times w$ between two vectors v and w.

Problem 3: Angle between Vectors

Given are two vectors

$$a = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 21 \\ x \end{bmatrix}$$

How do we have to choose x, such that the vectors are perpendicular? Verify your solution with R

Solution

Two vectors are perpendicular, if their dot product is 0. The dot product between a and b is computed as

$$a \cdot b = 8 * 2 + 0 * 21 + 4 * x = 0$$

The above formula is an equation for the unknown x. Solving for x gives

$$x = -\frac{8 * 2}{4} = -4$$

In R:

```
a <- c(8, 0, 4)
x <- -4
b <- c(2, 21, x )
crossprod(a, b)
```

```
##      [,1]
## [1,]    0
```

Problem 4: Phenotypes and Genotypes

Farmer Frank Miller has two cows named `Delilah` and `Rosy`. Delilah has completed 8 lactations with the following results

Lactation	Milk.Yield
1	5852
2	6833
3	7984
4	7869
5	7322
6	8216
7	8622
8	7851

Rosy is a young cow and has completed just 2 lactation which is shown below.

Lactation	Milk.Yield
1	6249
2	7312

Your Tasks

a. Compute for both cows the sum and the mean of all lactation results using R. **Hint**: Have a look at the functions `mean()` and `sum()` in R.
b. Our farmer wants to know which of the two cows would be a better mother for his breeding herd when looking at the traits **milk yield** and **longevity**. The trait longevity is defined as the number of years a cow is able to produce milk. From the breeding association the farmer receives the following predicted breeding values for the two cows. Please explain which of the two cows is the better choice as a mother.

Cows	Milk.Yield	Longevity
Delilah	-1037	112
Rosy	471	122

Solution

a. Assuming the milk performances are assigned the following vector

```
# milk yield of Delilah
vec_lact_perf_Delilah <- c(5852, 6833, 7984, 7869, 7322, 8216, 8622, 7851)
```

Then the total milk yield for Delilah is computed as

```
sum(vec_lact_perf_Delilah)
```

```
## [1] 60549
```

and the mean milk yield is computed as

```r
mean(vec_lact_perf_Delilah)
```

```
## [1] 7568.625
```

The same computations can be done for Rosy

```r
# milk yield of Rosy
vec_lact_perf_Rosy <- c(6249, 7312)
```

```r
sum(vec_lact_perf_Rosy)
```

```
## [1] 13561
```

```r
mean(vec_lact_perf_Rosy)
```

```
## [1] 6780.5
```

b. Parents do not pass their phenotypes to their offspring, but just a random sample of their alleles. Because the predicted breeding values are an estimated of the genetic potential that is passed from a parent to an offspring it is better to rank the candidates based on their predicted breeding values. Because Rosy has higher breeding values in both traits, our farmer should select her as a mother to produce the next generation. Delilah is a good producing cow and therefore she should be kept in the herd. But it might be better to inseminate her with a beef sire to produce a fattening calf.