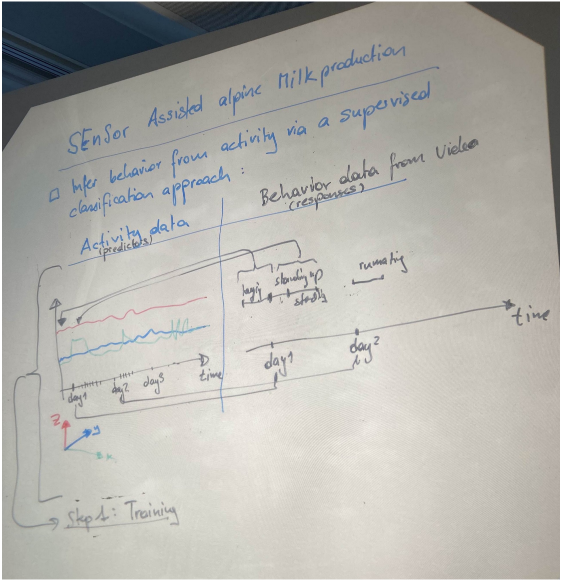
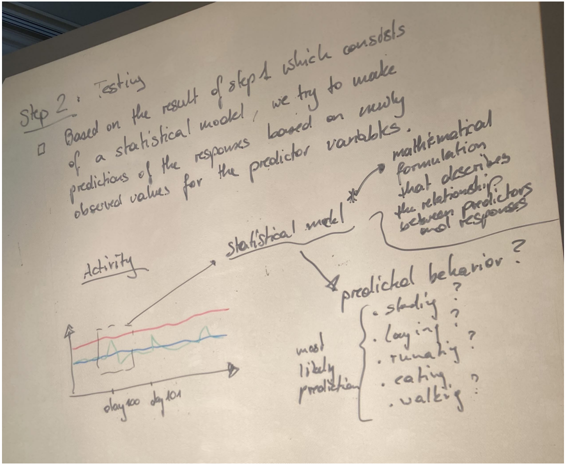
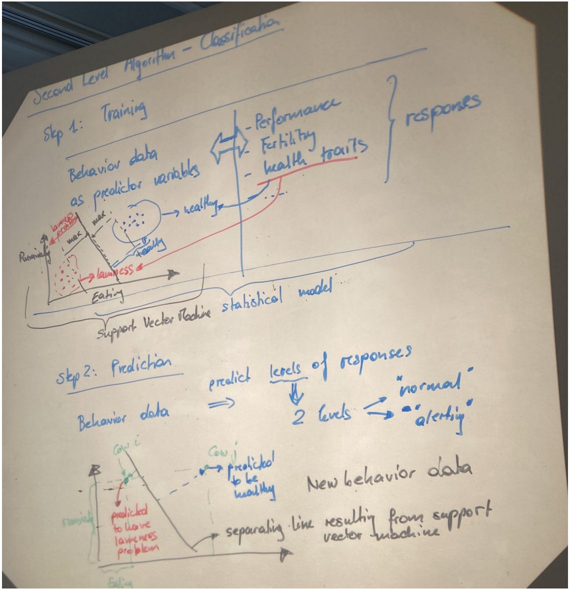
# OHP Picture 1



# OHP Picture 2



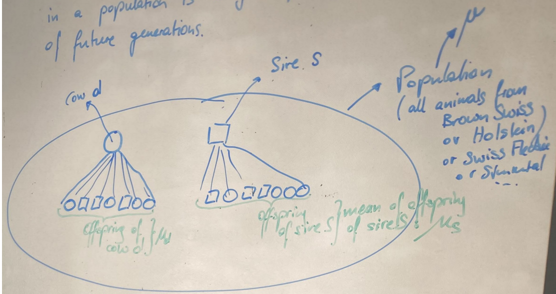
# OHP Picture 3



# OHP Picture 4

Breeding Value

- Measure that indicates whether a specific animal in a population is a "good" parent for offspring of future generations.

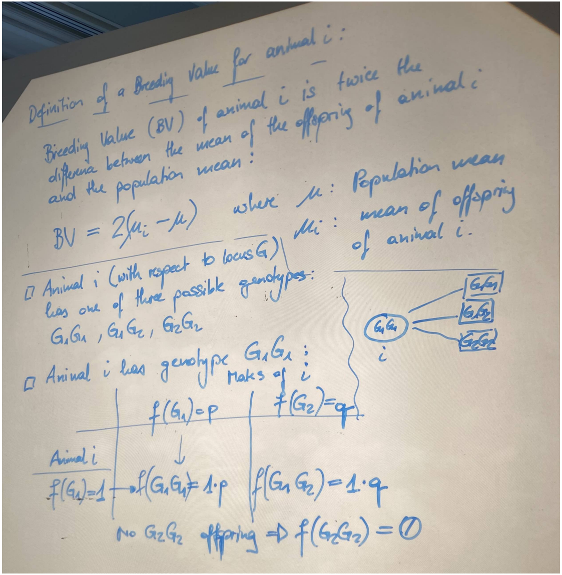



- Cow d (or sire S) is a "good" parents, if the mean of their offspring is greater than the population mean.

$\Rightarrow$ Compare $\mu_d$ to $\mu$, if $\mu_d > \mu \Rightarrow$ d is a good parent

$\mu_s$ to $\mu$, if $\mu_s > \mu \Rightarrow$ S is a good parent

# OHP Picture 5



Definition of a Breeding Value for animal $i$:

Breeding Value (BV) of animal $i$ is twice the difference between the mean of the offspring of animal $i$ and the population mean:

$$BV = 2(\mu_i - \mu)$$

where $\mu$: Population mean
$\mu_i$: mean of offspring of animal $i$.

□ Animal $i$ (with respect to locus $G$) has one of three possible genotypes: $G_1G_1$, $G_1G_2$, $G_2G_2$

$G_1G_1$ ($i$) → $G_1G_1$, $G_1G_2$, $G_2G_2$

□ Animal $i$ has genotype $G_1G_1$:

| | Mates of $i$: $f(G_1)=p$ | $f(G_2)=q$ |
|---|---|---|
| Animal $i$: $f(G_1)=1$ | $f(G_1G_1)=1\cdot p$ | $f(G_1G_2)=1\cdot q$ |

No $G_2G_2$ offspring $\Rightarrow f(G_2G_2)=0$

# OHP Picture 6

□ Mean of offspring of animal i with Genotype $G_1G_1$ is computed as an expected value:

$$\mu_{11} = p \cdot a + q \cdot d$$

(Mean of offspring of animal i with genotype $G_1G_1$)

$$\mu = (p-q)a + 2pqd$$

Breeding Value ($BV_{11}$) for animal i with genotype $G_1G_1$:

$$BV_{11} = 2(\mu_{11} - \mu)$$

$$= 2(p \cdot a + q \cdot d - [(p-q)a + 2pqd])$$

US: Predicted Transmission Ability (PTA)

$$PTA = \frac{1}{2} BV = \mu_{11} - \mu$$

$$= 2(p \cdot a + q \cdot d - (p-q)a - 2pqd)$$

$$= 2(\cancel{p \cdot a} + q \cdot d \cancel{- p \cdot a} + q \cdot a - 2pqd)$$

$$= 2(q \cdot d + q \cdot a - 2pq \cdot d)$$

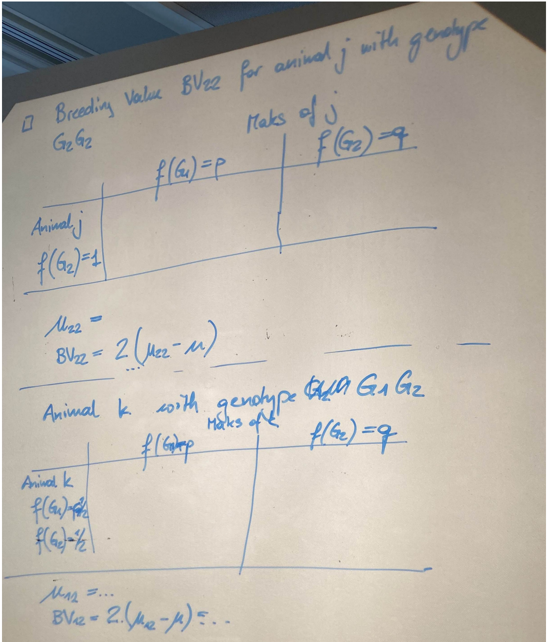
$$= 2(qa + qd(1-2pq))$$

$$= 2q(a + d(1-2p))$$

$$= 2q(a + (q-p)d)$$

$$p + q - 2p = q - p$$

# OHP Picture 7

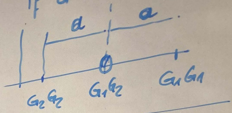


# OHP Picture 8

| | BV |
|---|---|
| $G_1G_1$ | $2q\alpha$ |
| $G_1G_2$ | $(q-p)\alpha$ |
| $G_2G_2$ | $-2p\alpha$ |

with $\alpha = a + (q-d)d$

if $d = 0 \Rightarrow \alpha = a$




- Allele Substitution
  - Given animal $i$ with genotype $G_2G_2$
  - Assume Gene Editing to replace one $G_2$ allele by a $G_1$ allele. After the GE animal $i$ has genotype $G_1G_2$
  - What happens to the BV of animal $i$
  - Before GE : $BV_{22} = -2p\alpha$
  - After GE : $BV_{12} = (q-p)\alpha$

$\Delta = BV_{12} - BV_{22}$

$\Rightarrow \Delta = (q-p)\alpha - (-2p\alpha) = q\alpha - p\alpha + 2p\alpha$

$= q\alpha + p\alpha = (p+q)\alpha = \underline{\alpha}$

- Again replace $G_2$ by $G_1$ in animal $i \Rightarrow G_1G_1$

$\Delta = BV_{11} - BV_{12} = 2q\alpha - (q-p)\alpha = \underline{\alpha}$

# OHP Picture 9

□ Genotypic Values $V_{ij}$ for animal with genotype $G_iG_j$

| Genotype | $V_{ij}$ | $BV_{ij}$ | Difference $V_{ij} - BV_{ij}$ |
|---|---|---|---|
| $G_1G_1$ | $a$ | $2q\alpha$ | $a - 2q\alpha = \mu - 2q^2d$ |
| $G_1G_2$ | $d$ | $(q-p)\alpha$ | $d - (q-p)\alpha$ |
| $G_2G_2$ | $-a$ | $-2p\alpha$ | $-a - (-2p\alpha)$ |

$$
\begin{aligned}
V_{11} - BV_{11} &= a - 2q\alpha \\
&= a - 2q[a + (q-p)d] \\
&= a - 2qa - 2q^2d + 2pqd \\
&= a(1-2q) - 2q^2d + 2pqd \\
&= \underbrace{[(p-q)a + 2pqd]}_{\mu} - 2q^2d \\
&= \mu + D_{11} \quad \text{where } \underbrace{D_{11} = -2q^2d}_{\text{Dominance deviation of } G_1G_1}
\end{aligned}
$$

# OHP Picture 10

□ Decomposition of the genotypic value

$V_{11} - BV_{11} = \mu + D_{11}$

$V_{12} - BV_{12} = \mu + D_{12}$

$V_{22} - BV_{22} = \mu + D_{22}$

$\Rightarrow$ In general: $V_{ij} - BV_{ij} = \mu + D_{ij}$

$$\boxed{V_{ij} = \mu + BV_{ij} + D_{ij}}$$

# OHP Picture 11

A further statistical quantity to describe variation of a random Variable (V) is the variance

- Variance is the second moment of random variable
- For discrete random variable $V$:

$$Var[V] = \sum_{i \in \{G_1G_1, G_1G_2, G_2G_2\}} f(i) \cdot (i-\mu)^2$$

$$\begin{aligned} &= (V_{11}-\mu)^2 \cdot f(G_1G_1) \\ &\quad + (V_{12}-\mu)^2 \cdot f(G_1G_2) \\ &\quad + (V_{22}-\mu)^2 \cdot f(G_2G_2) \\ &= \ldots = 2pq\,a^2 + (2pqd)^2 \\ &= \underbrace{\sigma_A^2}_{\text{variance of BV}} + \sigma_D^2 \end{aligned}$$

$\sigma_D^2 \rightarrow$ variance of dominance deviation