Livestock Breeding and Genomics - Solution 3

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Problem 1: Own Performance

Given is the dataset with weight observations for 12 animals. The heritability (h^2) for the trait is 0.2025. The population mean μ can assumed to be the mean of the weights in the table below.

‘

Animal	Weight
1	285
2	282
3	278
4	280
5	281
6	282
7	285
8	282
9	281
10	287
11	281
12	282

Your Tasks

- Compute the breeding values for all animals given in the table above
- Compute the accuracies of the breeding values of all animals shown in the table above.

Solution

The predicted breeding value $\widehat{u_i}$ of animal i is computed as

$$\widehat{u_i} = h^2(y_i - \mu)$$

where h^2 is the heritability given in the problem, y_i is observation of animal i and μ is the poputlation mean. The population mean is to be computed from the mean of the observations. Hence

```
n_mu_weight <- mean(tbl_weight$Weight)
```

Problem 2: Breeding Value Prediction Based on Repeated Observations

```
geb_gew <- 52
mu2 <- 170
rep <- 0.65
```

```r
h2 <- 0.45
y <- 320
mu <- 250
```

Elsa has observations for her birth weight (52 kg) and some more repeated measures for her weight. We assume the heritability to be $h^2 = 0.45$. The population mean for the repeated observations of the weight is 170 kg. The repeatability of the weight measurements is $t = 0.65$.

The following tables contains all observed values for the weight.

```r
nr_measure <- 10
wean_weight <- y
slope <- (wean_weight-geb_gew)/(nr_measure-1)
measure <- c(1:nr_measure)
weight <- round(slope*(measure-1) + geb_gew, digits = 0)
mean_weight <- mean(weight)
dfWeightTable <- data.frame(Measurement = measure, Weight = weight)
knitr::kable(dfWeightTable)
```

Measurement	Weight
1	52
2	82
3	112
4	141
5	171
6	201
7	231
8	260
9	290
10	320

a) Predict the breeding value for Elsa based on the repeated weight records.
b) What is the reliability for the predicted breeding value from 2a)?
c) Compare the reliability from 2b) with the reliability that would result from a prediction of breeding values based on own performance.

Solution

a) The predicted breeding value based on repeated records is

```r
hat_a_rep_meas <- round((nr_measure * h2)/(1+(nr_measure - 1)*rep)*(mean_weight - mu2), digits = 2)
```

$$\hat{u}_i = \frac{nh^2}{1 + (n-1)t}(\bar{y}_i - \mu) = \frac{10 * 0.45}{1 + (9 * 0.65)}(186 - 170) = 10.51$$

b) The reliability for the predicted breeding value from 2a) is

```r
rel_rep_rec <- (nr_measure * h2)/(1+(nr_measure - 1)*rep)
```

$$B = r^2_{u,\bar{y}} = b = \frac{nh^2}{1 + (n-1)t} = \frac{10 * 0.45}{1 + (9 * 0.65)} = 0.66$$

c) The reliability of the predicted breeding values based on repeated records is larger than the reliability of the prediction based on one record. The relation between the two reliabilities is

$$\frac{r_{u,\bar{y}}^2}{r_{u,y}^2} = \frac{n}{1 + (n-1)t} = \frac{10}{1 + (9 * 0.65)} = 1.46$$

Problem 3: Predict Breeding Values Based on Progeny Records

```
n_nr_progeny <- 5
```

A few years later Elsa was the dam of 5 offspring. Each of the offspring has a record for weaning weight. Predict the breeding value of Elsa for weaning weight based on the offpsring records listed in the following table.

Offspring	Weaning Weight
1	320
2	319
3	320
4	320
5	321

The mean and the heritability can be taken the same as in Problems 1 and 2 resulting in

$h^2 = 0.45$ and $\mu = 250$

Solution

The predicted breeding value based on progeny records is defined as

$$\hat{u_i} = b * (\bar{y_i} - \mu) \tag{1}$$

where $\bar{y_i}$ corresponds to the mean of the progeny records for animal i, and b is the regression coefficient which can be shown to be

$$b = \frac{2n}{n + k}$$

where n is the number of offspring and k corresponds to

$$k = \frac{4 - h^2}{h^2}$$

Inserting the numbers given in the problem task results in

$$k = \frac{4 - 0.45}{0.45} = 7.89$$

Using the computed value of k allows to get the regression coefficient b.

$$b = \frac{2 * 5}{5 + 7.89} = 0.78$$

The predicted breeding value based on progeny records corresponds to

$$\hat{u_i} = 0.78 * (320 - 250) = 54.31$$