

Livestock Breeding and Genomics - Exercise 8

Peter von Rohr

2022-11-25

Problem 1: Inverse Numerator Relationship Matrix

During the lecture the method of computing the inverse numerator relationship matrix A^{-1} directly was introduced. The computation is based on the LDL-decomposition. As a result, we can write

$$A^{-1} = (L^T)^{-1} \cdot D^{-1} \cdot L^{-1}$$

where $L^{-1} = I - P$, and D^{-1} is a diagonal matrix with $(D^{-1})_{ii} * \sigma_u^{-2} = \text{var}(m_i)^{-1}$.

Tasks

- Use the example pedigree given below and compute the matrices L^{-1} and D^{-1} to compute A^{-1}
- Verify your result using the function `getAinv()` from package `pedigreemm`.

Pedigree

```
nr_animal <- 6
tbl_pedigree <- tibble::tibble(Calf = c(1:nr_animal),
                              Sire = c(NA, NA, NA, 1, 3, 4),
                              Dam = c(NA, NA, NA, 2, 2, 5))
tbl_pedigree
```

```
## # A tibble: 6 x 3
##   Calf Sire  Dam
##   <int> <dbl> <dbl>
## 1     1   NA   NA
## 2     2   NA   NA
## 3     3   NA   NA
## 4     4     1     2
## 5     5     3     2
## 6     6     4     5
```

Problem 2: Rules

The following diagram helps to illustrate the rules for constructing A^{-1}

		D^{-1}						L^{-1}																																																																																																																																																														
		<table style="border-collapse: collapse; width: 100%;"> <tr><td></td><td>[,1]</td><td>[,2]</td><td>[,3]</td><td>[,4]</td><td>[,5]</td><td>[,6]</td></tr> <tr><td>[1,]</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>[2,]</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>[3,]</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>[4,]</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>0</td></tr> <tr><td>[5,]</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td></tr> <tr><td>[6,]</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr> </table>							[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	0	0	0	[2,]	0	1	0	0	0	0	[3,]	0	0	1	0	0	0	[4,]	0	0	0	2	0	0	[5,]	0	0	0	0	2	0	[6,]	0	0	0	0	0	2	<table style="border-collapse: collapse; width: 100%;"> <tr><td></td><td>[,1]</td><td>[,2]</td><td>[,3]</td><td>[,4]</td><td>[,5]</td><td>[,6]</td></tr> <tr><td>[1,]</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr><td>[2,]</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr><td>[3,]</td><td>0.0</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0.0</td><td>0</td></tr> <tr><td>[4,]</td><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0.0</td><td>0</td></tr> <tr><td>[5,]</td><td>0.0</td><td>-0.5</td><td>-0.5</td><td>0.0</td><td>1.0</td><td>0</td></tr> <tr><td>[6,]</td><td>0.0</td><td>0.0</td><td>0.0</td><td>-0.5</td><td>-0.5</td><td>1</td></tr> </table>							[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.0	0.0	0.0	0.0	0.0	0	[2,]	0.0	1.0	0.0	0.0	0.0	0	[3,]	0.0	0.0	1.0	0.0	0.0	0	[4,]	-0.5	-0.5	0.0	1.0	0.0	0	[5,]	0.0	-0.5	-0.5	0.0	1.0	0	[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																							
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																																																
[1,]	1	0	0	0	0	0																																																																																																																																																																
[2,]	0	1	0	0	0	0																																																																																																																																																																
[3,]	0	0	1	0	0	0																																																																																																																																																																
[4,]	0	0	0	2	0	0																																																																																																																																																																
[5,]	0	0	0	0	2	0																																																																																																																																																																
[6,]	0	0	0	0	0	2																																																																																																																																																																
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																																																
[1,]	1.0	0.0	0.0	0.0	0.0	0																																																																																																																																																																
[2,]	0.0	1.0	0.0	0.0	0.0	0																																																																																																																																																																
[3,]	0.0	0.0	1.0	0.0	0.0	0																																																																																																																																																																
[4,]	-0.5	-0.5	0.0	1.0	0.0	0																																																																																																																																																																
[5,]	0.0	-0.5	-0.5	0.0	1.0	0																																																																																																																																																																
[6,]	0.0	0.0	0.0	-0.5	-0.5	1																																																																																																																																																																
		<table style="border-collapse: collapse; width: 100%;"> <tr><td></td><td>[,1]</td><td>[,2]</td><td>[,3]</td><td>[,4]</td><td>[,5]</td><td>[,6]</td></tr> <tr><td>[1,]</td><td>1</td><td>0</td><td>0</td><td>-0.5</td><td>0.0</td><td>0.0</td></tr> <tr><td>[2,]</td><td>0</td><td>1</td><td>0</td><td>-0.5</td><td>-0.5</td><td>0.0</td></tr> <tr><td>[3,]</td><td>0</td><td>0</td><td>1</td><td>0.0</td><td>-0.5</td><td>0.0</td></tr> <tr><td>[4,]</td><td>0</td><td>0</td><td>0</td><td>1.0</td><td>0.0</td><td>-0.5</td></tr> <tr><td>[5,]</td><td>0</td><td>0</td><td>0</td><td>0.0</td><td>1.0</td><td>-0.5</td></tr> <tr><td>[6,]</td><td>0</td><td>0</td><td>0</td><td>0.0</td><td>0.0</td><td>1.0</td></tr> </table>							[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-0.5	0.0	0.0	[2,]	0	1	0	-0.5	-0.5	0.0	[3,]	0	0	1	0.0	-0.5	0.0	[4,]	0	0	0	1.0	0.0	-0.5	[5,]	0	0	0	0.0	1.0	-0.5	[6,]	0	0	0	0.0	0.0	1.0	<table style="border-collapse: collapse; width: 100%;"> <tr><td></td><td>[,1]</td><td>[,2]</td><td>[,3]</td><td>[,4]</td><td>[,5]</td><td>[,6]</td></tr> <tr><td>[1,]</td><td>1</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td></tr> <tr><td>[2,]</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td></tr> <tr><td>[3,]</td><td>0</td><td>0</td><td>1</td><td>0</td><td>-1</td><td>0</td></tr> <tr><td>[4,]</td><td>0</td><td>0</td><td>0</td><td>2</td><td>0</td><td>-1</td></tr> <tr><td>[5,]</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td><td>-1</td></tr> <tr><td>[6,]</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>2</td></tr> </table>							[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1	0	0	-1	0	0	[2,]	0	1	0	-1	-1	0	[3,]	0	0	1	0	-1	0	[4,]	0	0	0	2	0	-1	[5,]	0	0	0	0	2	-1	[6,]	0	0	0	0	0	2	<table style="border-collapse: collapse; width: 100%;"> <tr><td></td><td>[,1]</td><td>[,2]</td><td>[,3]</td><td>[,4]</td><td>[,5]</td><td>[,6]</td></tr> <tr><td>[1,]</td><td>1.5</td><td>0.5</td><td>0.0</td><td>-1.0</td><td>0.0</td><td>0</td></tr> <tr><td>[2,]</td><td>0.5</td><td>2.0</td><td>0.5</td><td>-1.0</td><td>-1.0</td><td>0</td></tr> <tr><td>[3,]</td><td>0.0</td><td>0.5</td><td>1.5</td><td>0.0</td><td>-1.0</td><td>0</td></tr> <tr><td>[4,]</td><td>-1.0</td><td>-1.0</td><td>0.0</td><td>2.5</td><td>0.5</td><td>-1</td></tr> <tr><td>[5,]</td><td>0.0</td><td>-1.0</td><td>-1.0</td><td>0.5</td><td>2.5</td><td>-1</td></tr> <tr><td>[6,]</td><td>0.0</td><td>0.0</td><td>0.0</td><td>-1.0</td><td>-1.0</td><td>2</td></tr> </table>							[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[1,]	1.5	0.5	0.0	-1.0	0.0	0	[2,]	0.5	2.0	0.5	-1.0	-1.0	0	[3,]	0.0	0.5	1.5	0.0	-1.0	0	[4,]	-1.0	-1.0	0.0	2.5	0.5	-1	[5,]	0.0	-1.0	-1.0	0.5	2.5	-1	[6,]	0.0	0.0	0.0	-1.0	-1.0	2
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																																																
[1,]	1	0	0	-0.5	0.0	0.0																																																																																																																																																																
[2,]	0	1	0	-0.5	-0.5	0.0																																																																																																																																																																
[3,]	0	0	1	0.0	-0.5	0.0																																																																																																																																																																
[4,]	0	0	0	1.0	0.0	-0.5																																																																																																																																																																
[5,]	0	0	0	0.0	1.0	-0.5																																																																																																																																																																
[6,]	0	0	0	0.0	0.0	1.0																																																																																																																																																																
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																																																
[1,]	1	0	0	-1	0	0																																																																																																																																																																
[2,]	0	1	0	-1	-1	0																																																																																																																																																																
[3,]	0	0	1	0	-1	0																																																																																																																																																																
[4,]	0	0	0	2	0	-1																																																																																																																																																																
[5,]	0	0	0	0	2	-1																																																																																																																																																																
[6,]	0	0	0	0	0	2																																																																																																																																																																
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]																																																																																																																																																																
[1,]	1.5	0.5	0.0	-1.0	0.0	0																																																																																																																																																																
[2,]	0.5	2.0	0.5	-1.0	-1.0	0																																																																																																																																																																
[3,]	0.0	0.5	1.5	0.0	-1.0	0																																																																																																																																																																
[4,]	-1.0	-1.0	0.0	2.5	0.5	-1																																																																																																																																																																
[5,]	0.0	-1.0	-1.0	0.5	2.5	-1																																																																																																																																																																
[6,]	0.0	0.0	0.0	-1.0	-1.0	2																																																																																																																																																																
		$(L^T)^{-1}$						$(L^T)^{-1} \cdot D^{-1}$						A^{-1}																																																																																																																																																								

Tasks

- Go through the list of animals in the pedigree and write down the contributions that are made to the different elements of matrix A^{-1}
- Based on the different contributions, try to come up with some general rules

Problem 3: Program using the Rules

Write a program in R that implements the rules found in the solution of Problem 2. Test your program with the pedigree given in Problem 1. Compare the results that you obtain with the result obtained from the function `pedigreemm::getAinv()`.