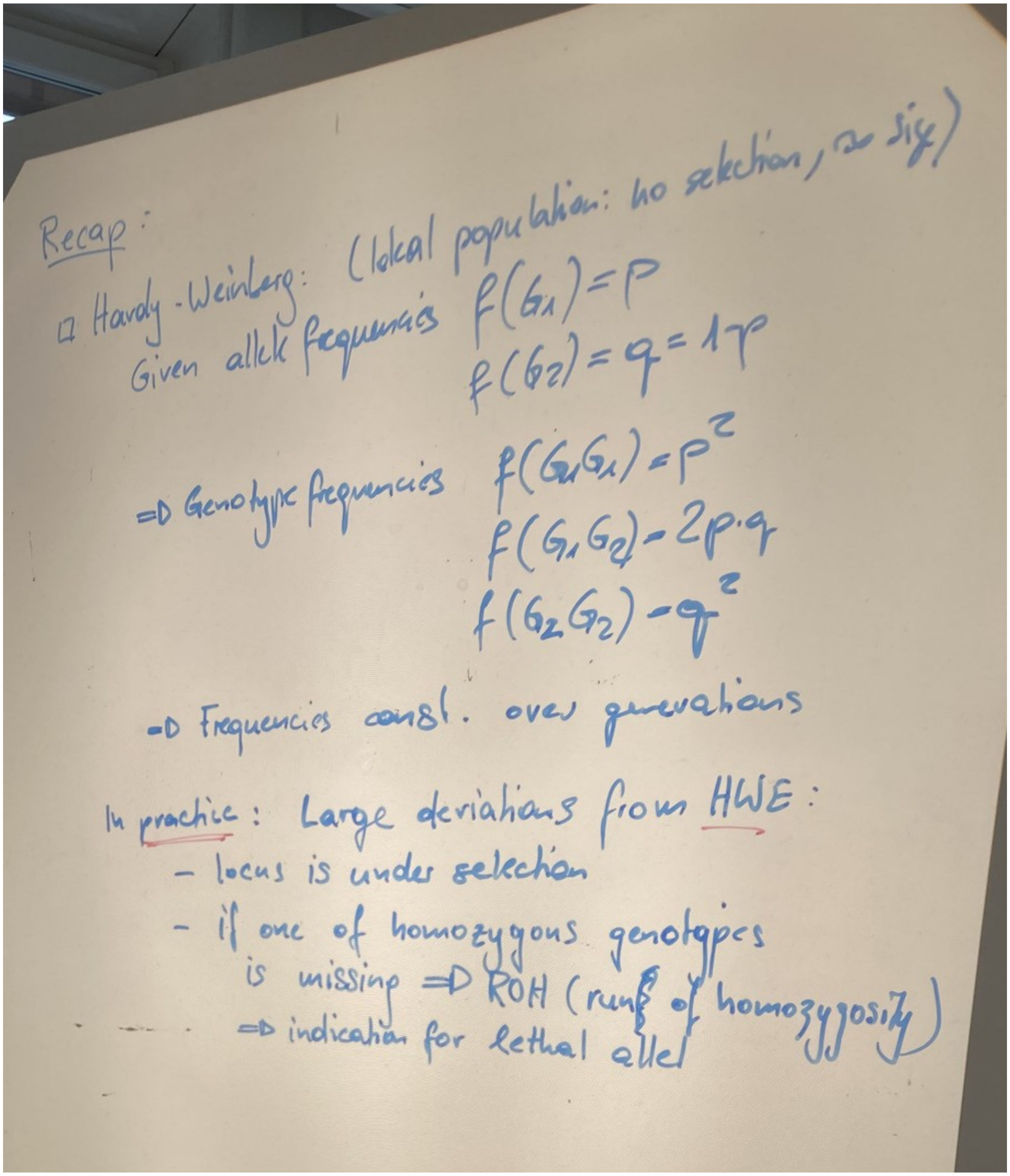
OHP Picture 1



Recap:

☐ Hardy-Weinberg: (Ideal population: no selection, ∞ size)

Given allele frequencies $f(G_1) = p$

$f(G_2) = q = 1-p$

⇒ Genotype frequencies $f(G_1G_1) = p^2$

$f(G_1G_2) = 2p \cdot q$

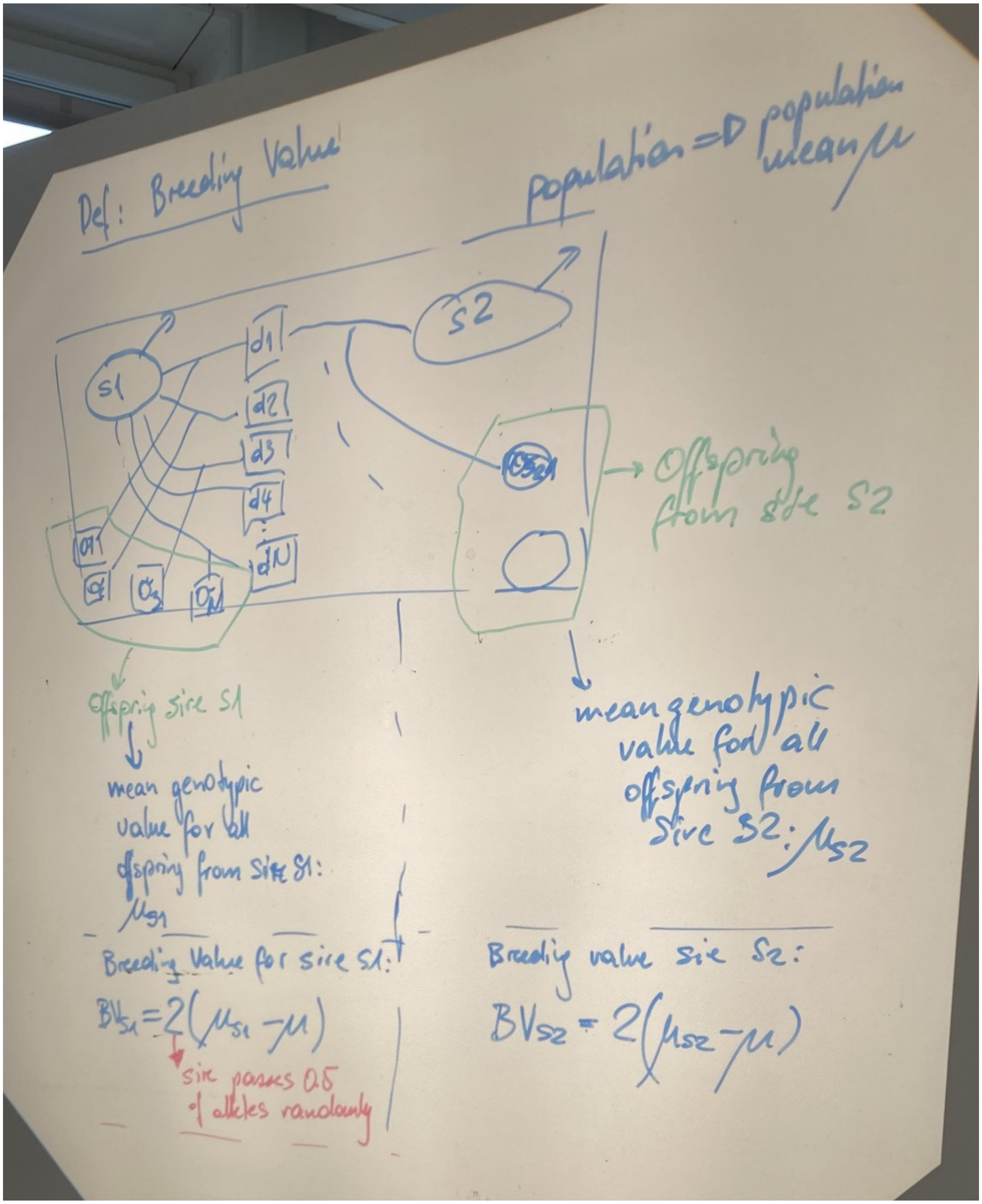
$f(G_2G_2) = q^2$

⇒ Frequencies const. over generations

In practice: Large deviations from HWE:

- locus is under selection
- if one of homozygous genotypes is missing ⇒ ROH (runs of homozygosity)
  ⇒ indication for lethal allel

OHP Picture 2



Def: Breeding Value

population ⇒ population mean μ

Offspring from sire S2

offspring sire S1

mean genotypic value for all offspring from sire S1: μ_{S1}

mean genotypic value for all offspring from sire S2: μ_{S2}

Breeding Value for sire S1:

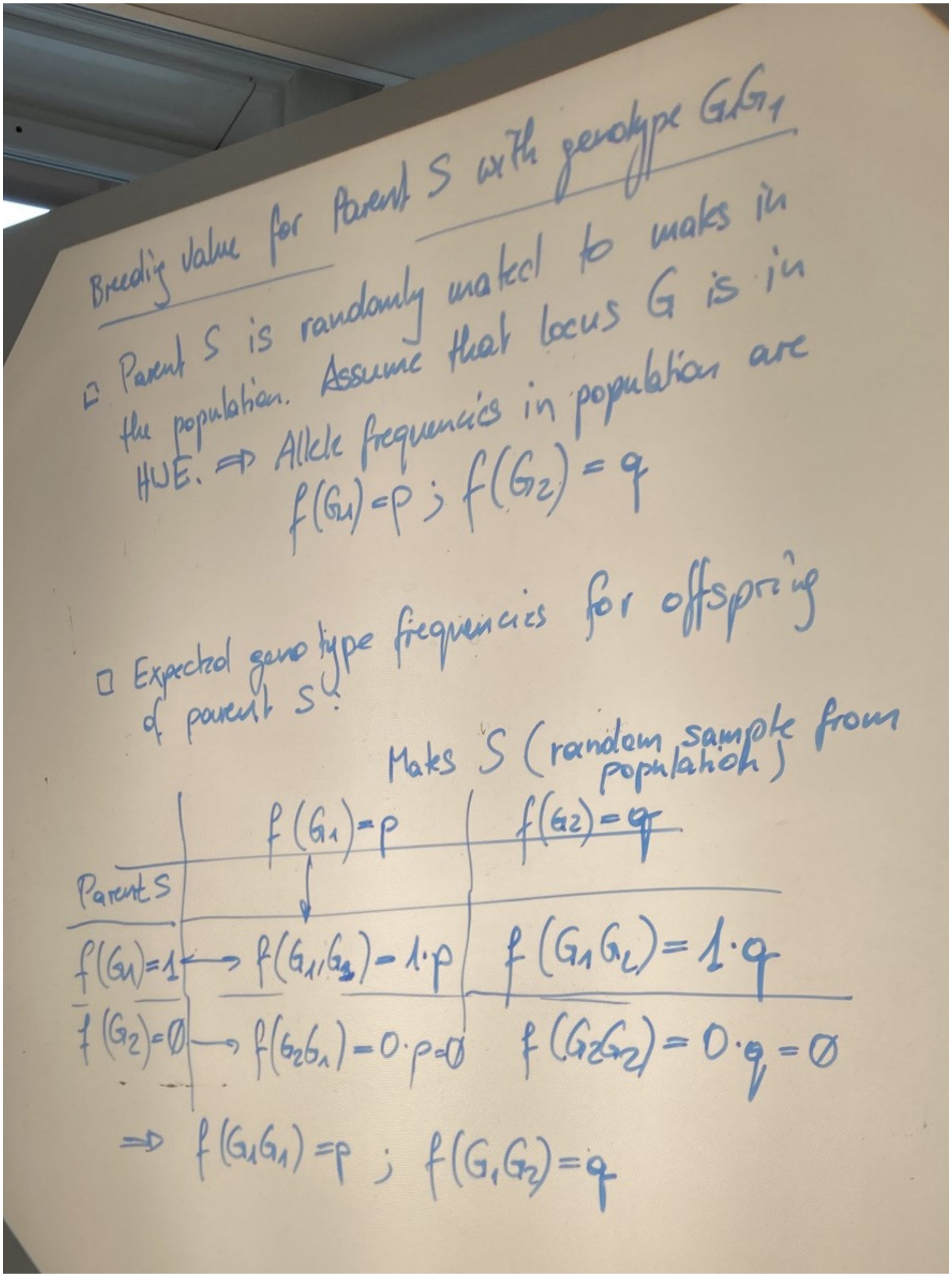
$$BV_{S1} = 2(\mu_{S1} - \mu)$$

sire passes 0.5 of alleles randomly

Breeding value sire S2:

$$BV_{S2} = 2(\mu_{S2} - \mu)$$

OHP Picture 3



Breeding value for Parent S with genotype G_1G_1

☐ Parent S is randomly mated to mates in the population. Assume that locus G is in HWE. ⇒ Allele frequencies in population are

$$f(G_1)=p \; ; \; f(G_2)=q$$

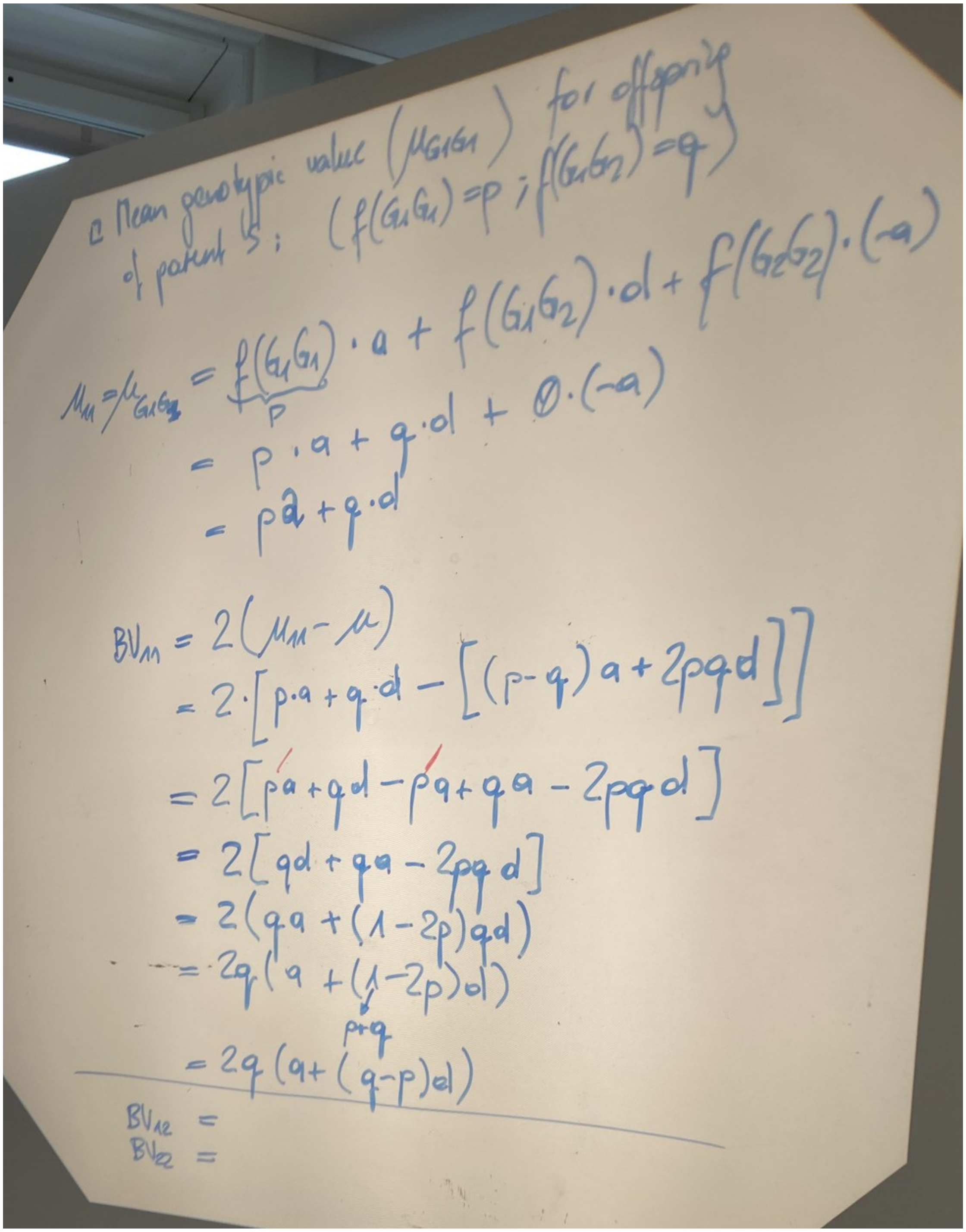
☐ Expected genotype frequencies for offspring of parent S?

Mates S (random sample from population)

Parent S	$f(G_1)=p$	$f(G_2)=q$
$f(G_1)=1$	$f(G_1,G_1)=1\cdot p$	$f(G_1G_2)=1\cdot q$
$f(G_2)=0$	$f(G_2G_1)=0\cdot p=0$	$f(G_2G_2)=0\cdot q=0$

$$\Rightarrow f(G_1G_1)=p \; ; \; f(G_1G_2)=q$$

OHP Picture 4



☐ Mean genotypic value ($\mu_{G_1G_1}$) for offspring of parent is: ($f(G_1G_1)=p$; $f(G_1G_2)=q$)

$$\mu_{11}=\mu_{G_1G_1}=\underbrace{f(G_1G_1)}_{p}\cdot a+f(G_1G_2)\cdot d+f(G_2G_2)\cdot(-a)$$

$$=p\cdot a+q\cdot d+0\cdot(-a)$$

$$=pa+q\cdot d$$

$$BV_{11}=2(\mu_{11}-\mu)$$

$$=2\cdot\left[p\cdot a+q\cdot d-\left[(p-q)a+2pqd\right]\right]$$

$$=2\left[\cancel{p}a+qd-\cancel{p}a+qa-2pqd\right]$$

$$=2\left[qd+qa-2pqd\right]$$

$$=2(qa+(1-2p)qd)$$

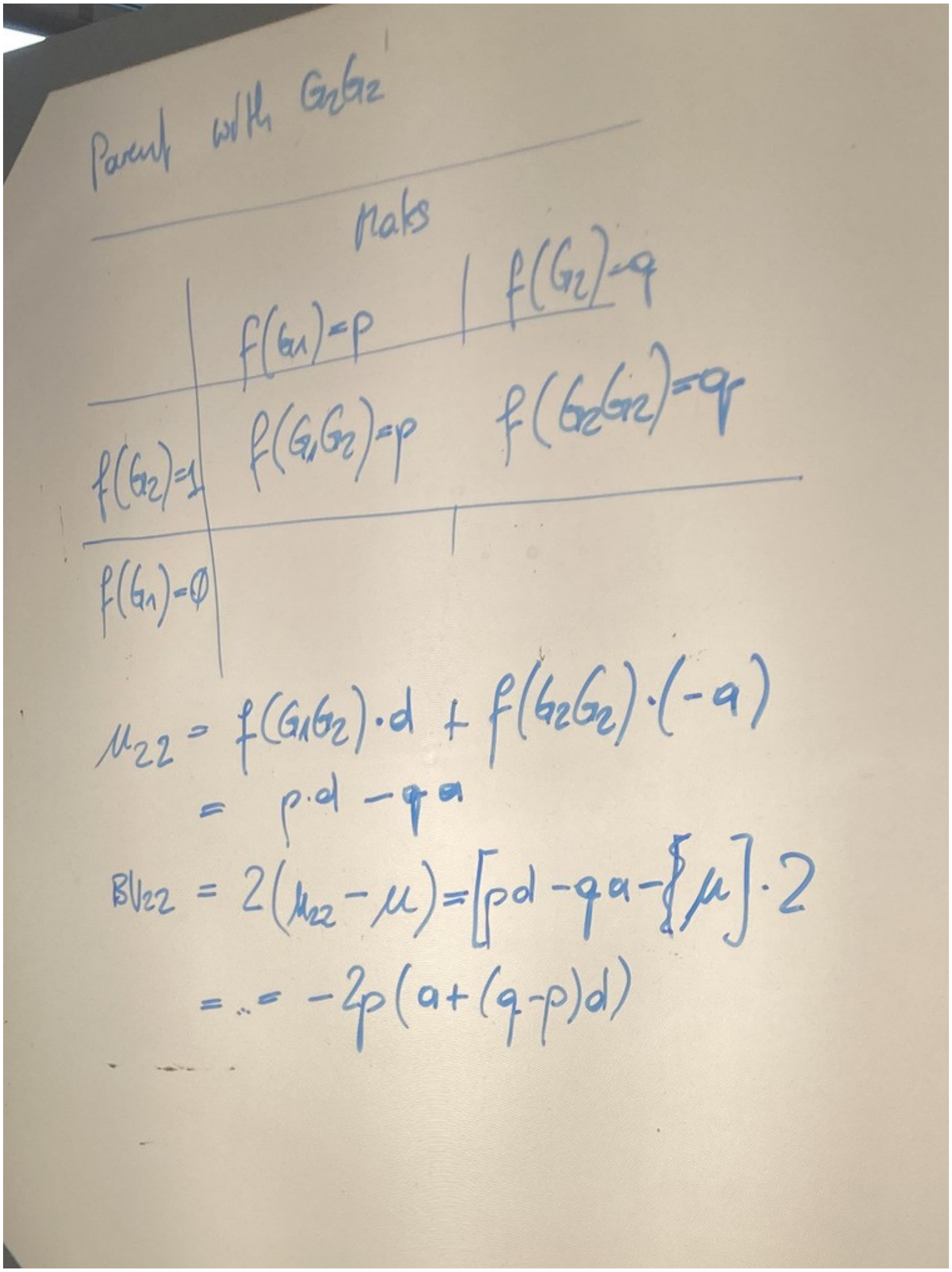
$$=2q(a+(\underset{\substack{\downarrow\\p+q}}{1}-2p)d)$$

$$=2q(a+(q-p)d)$$

$BV_{12}=$

$BV_{22}=$

OHP Picture 5



Parent with G_2G_2

	Males	
	$f(G_1)=p$	$f(G_2)=q$
$f(G_2)=1$	$f(G_1G_2)=p$	$f(G_2G_2)=q$
$f(G_1)=0$		

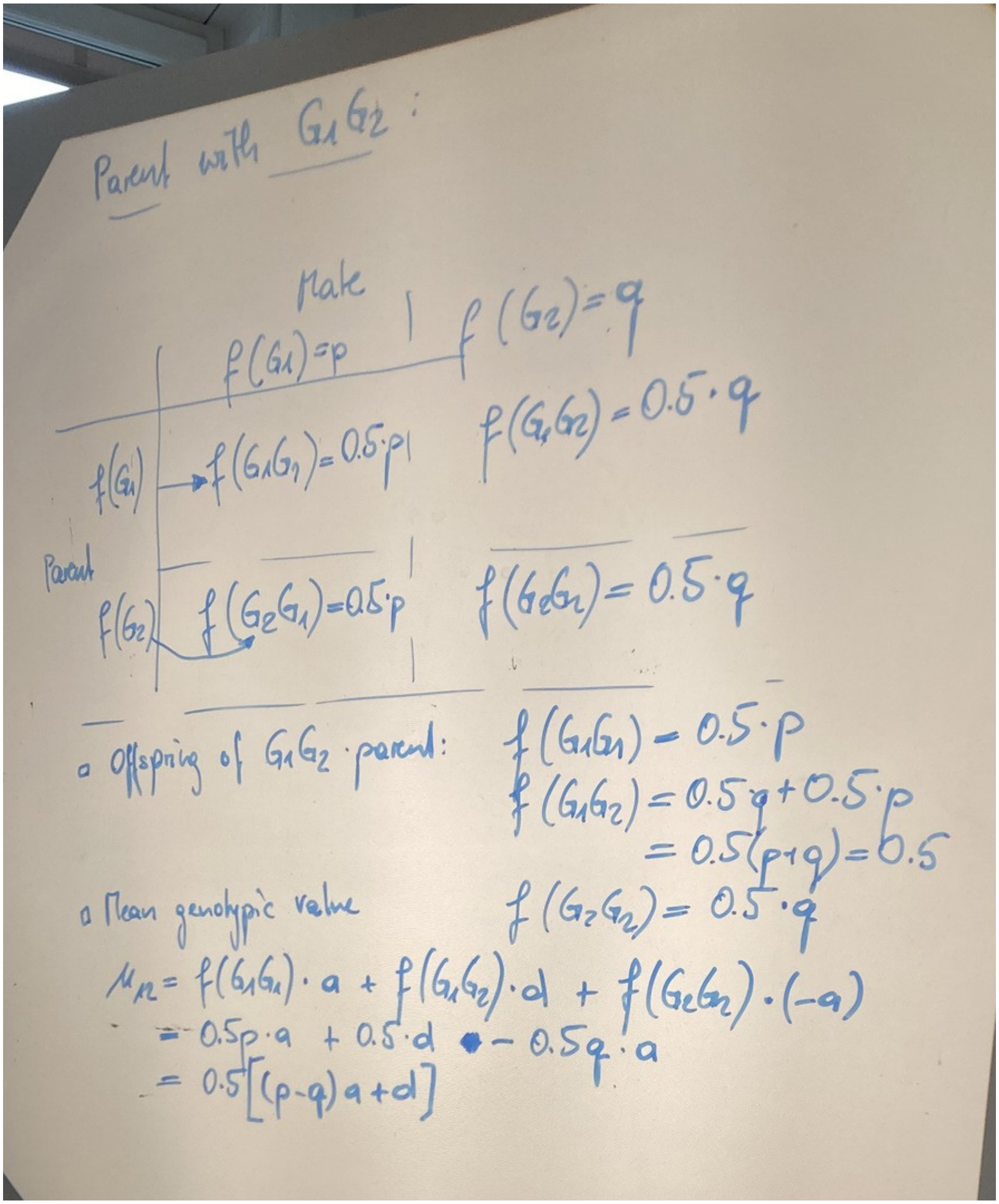
$$\mu_{22} = f(G_1G_2)\cdot d + f(G_2G_2)\cdot(-a)$$

$$= p\cdot d - qa$$

$$BV_{22} = 2(\mu_{22}-\mu) = [pd - qa - \{\mu\}]\cdot 2$$

$$= \ldots = -2p(a+(q-p)d)$$

OHP Picture 6



Parent with G_1G_2:

		Male $f(G_1)=p$	$f(G_2)=q$
Parent	$f(G_1)$	$f(G_1G_1)=0.5\cdot p$	$f(G_1G_2)=0.5\cdot q$
	$f(G_2)$	$f(G_2G_1)=0.5\cdot p$	$f(G_2G_2)=0.5\cdot q$

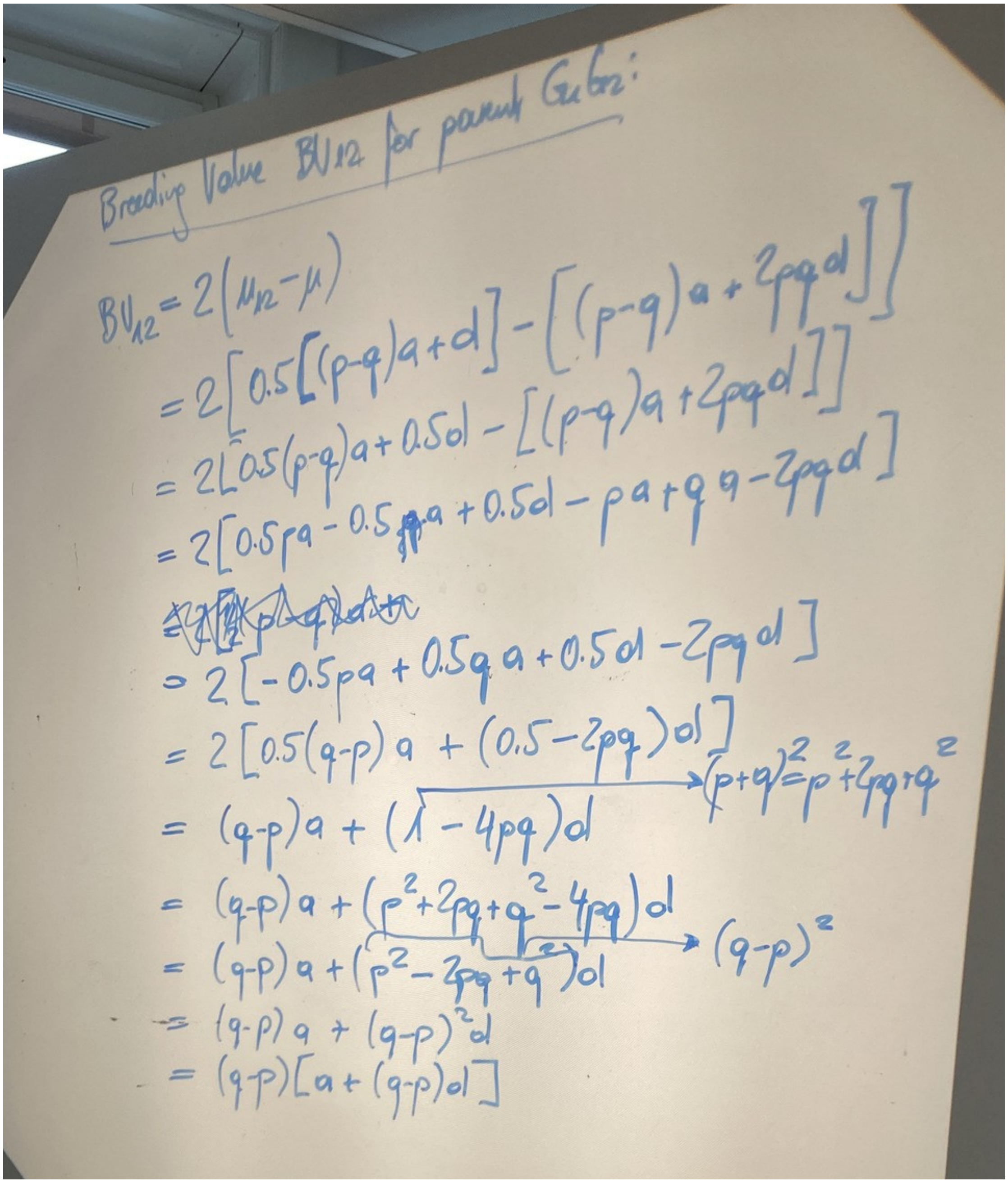
- Offspring of G_1G_2 · parent:

$$f(G_1G_1)=0.5\cdot p$$
$$f(G_1G_2)=0.5q+0.5\cdot p$$
$$=0.5(p+q)=0.5$$
$$f(G_2G_2)=0.5\cdot q$$

- Mean genotypic value

$$\mu_{12}=f(G_1G_1)\cdot a+f(G_1G_2)\cdot d+f(G_2G_2)\cdot(-a)$$
$$=0.5p\cdot a+0.5\cdot d-0.5q\cdot a$$
$$=0.5\left[(p-q)a+d\right]$$

OHP Picture 7



Breeding Value BV_{12} for parent G_1G_2:

$$BV_{12} = 2(\mu_{12} - \mu)$$

$$= 2\left[0.5[(p-q)a + d] - [(p-q)a + 2pqd]\right]$$

$$= 2[0.5(p-q)a + 0.5d - [(p-q)a + 2pqd]]$$

$$= 2[0.5pa - 0.5qa + 0.5d - pa + qa - 2pqd]$$

$$= 2[-0.5pa + 0.5qa + 0.5d - 2pqd]$$

$$= 2[0.5(q-p)a + (0.5 - 2pq)d]$$

$$= (q-p)a + (1 - 4pq)d \quad \rightarrow (p+q)^2 = p^2 + 2pq + q^2$$

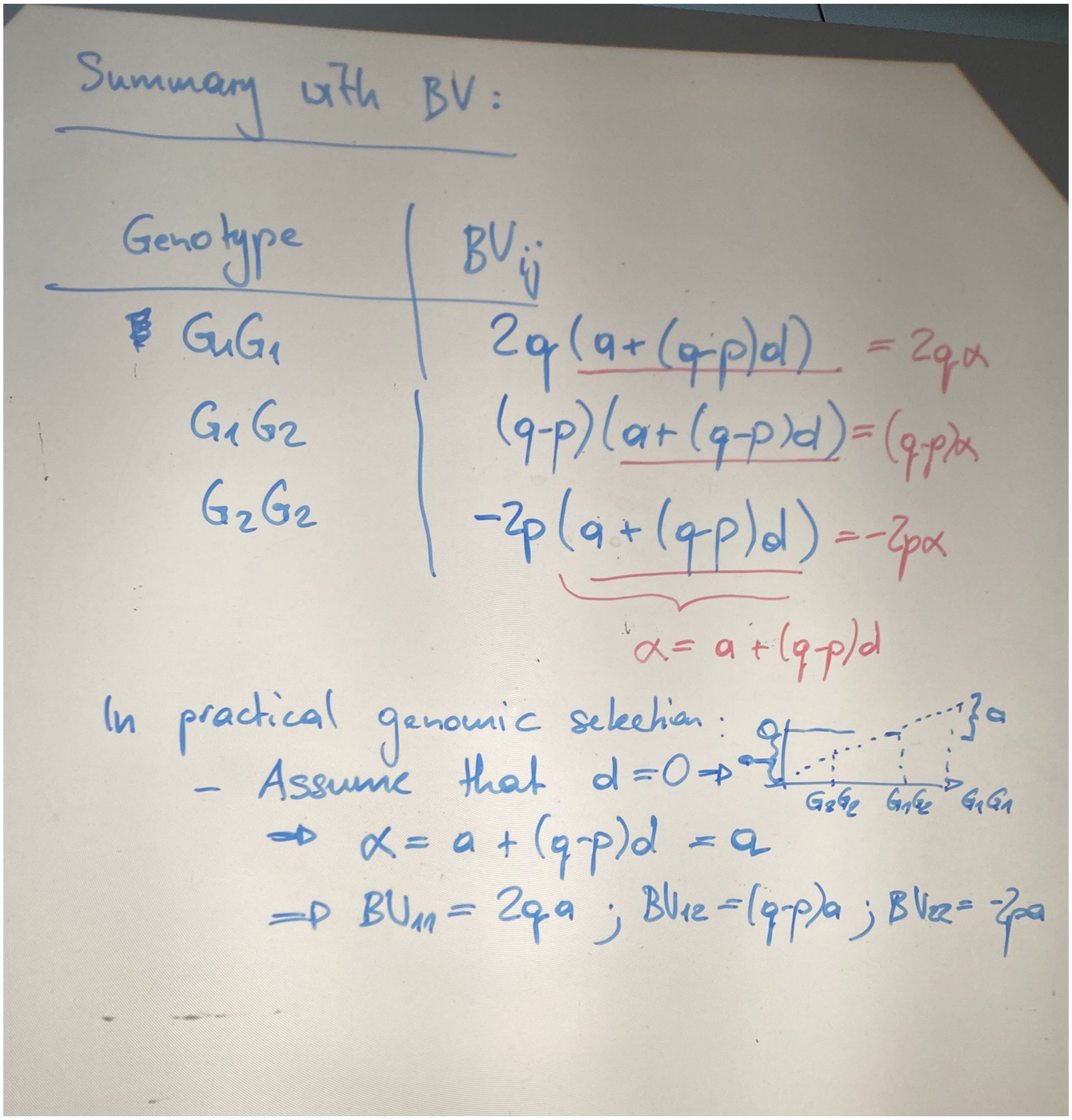
$$= (q-p)a + (p^2 + 2pq + q^2 - 4pq)d$$

$$= (q-p)a + (\underbrace{p^2 - 2pq + q^2}_{\rightarrow (q-p)^2})d$$

$$= (q-p)a + (q-p)^2 d$$

$$= (q-p)[a + (q-p)d]$$

OHP Picture 8



Summary with BV:

Genotype	BV_{ij}
G_1G_1	$2q(\underline{a+(q-p)d}) = 2q\alpha$
G_1G_2	$(q-p)(\underline{a+(q-p)d}) = (q-p)\alpha$
G_2G_2	$-2p(\underbrace{\underline{a+(q-p)d}}_{\alpha = a+(q-p)d}) = -2p\alpha$

In practical genomic selection:

- Assume that $d = 0 \Rightarrow$

$\Rightarrow \alpha = a + (q-p)d = a$

$\Rightarrow BV_{11} = 2q\,a \; ; \; BV_{12} = (q-p)a \; ; \; BV_{22} = -2pa$

OHP Picture 9