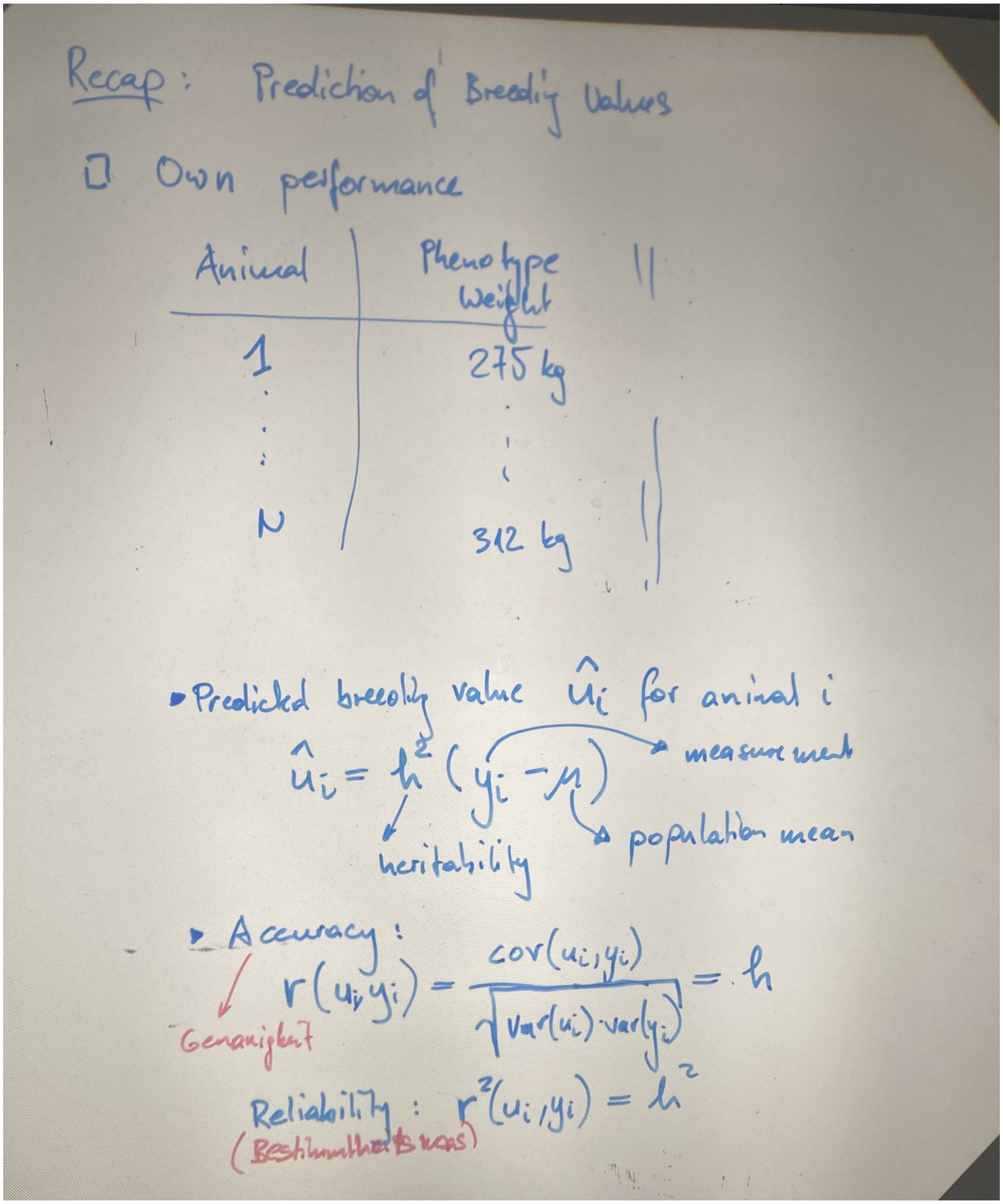
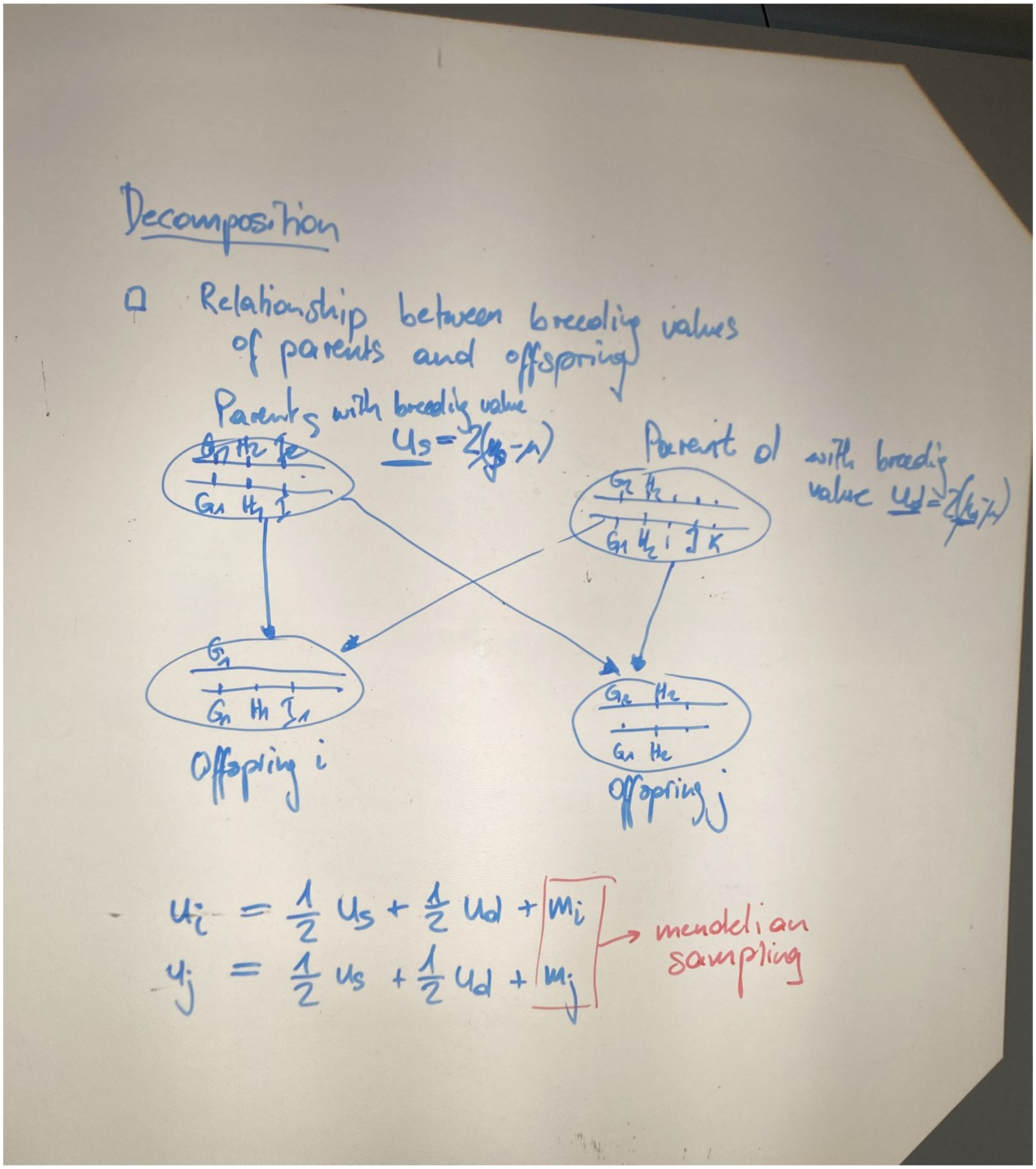
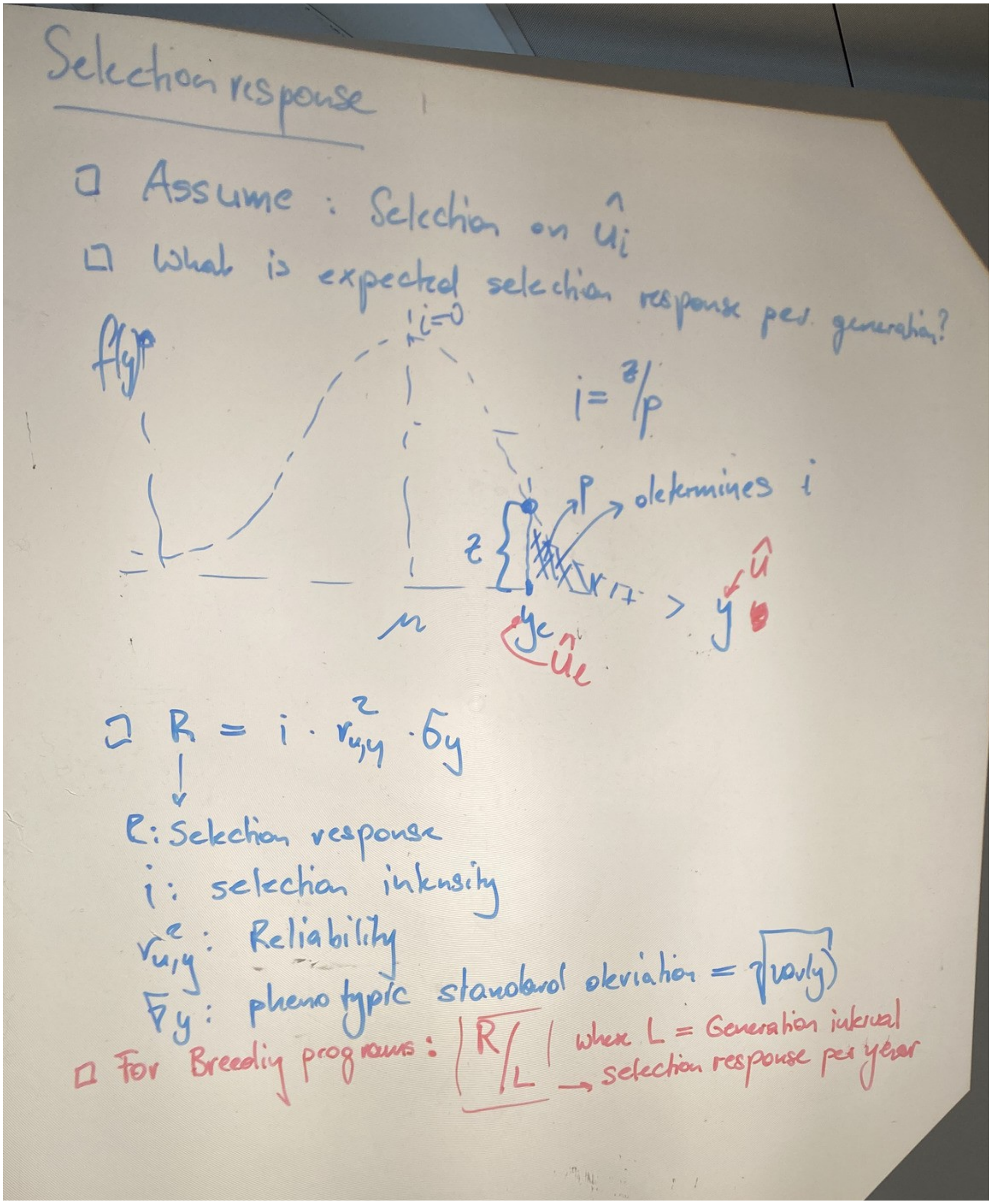
OHP Picture 1



OHP Picture 2



OHP Picture 3



OHP Picture 4

Pros and Cons of Own Performance

Pro: - simple to compute

Con: - only animals with phenotypic records get predicted ~~breeding~~ values
$\Rightarrow$ sex-limited traits (dairy)

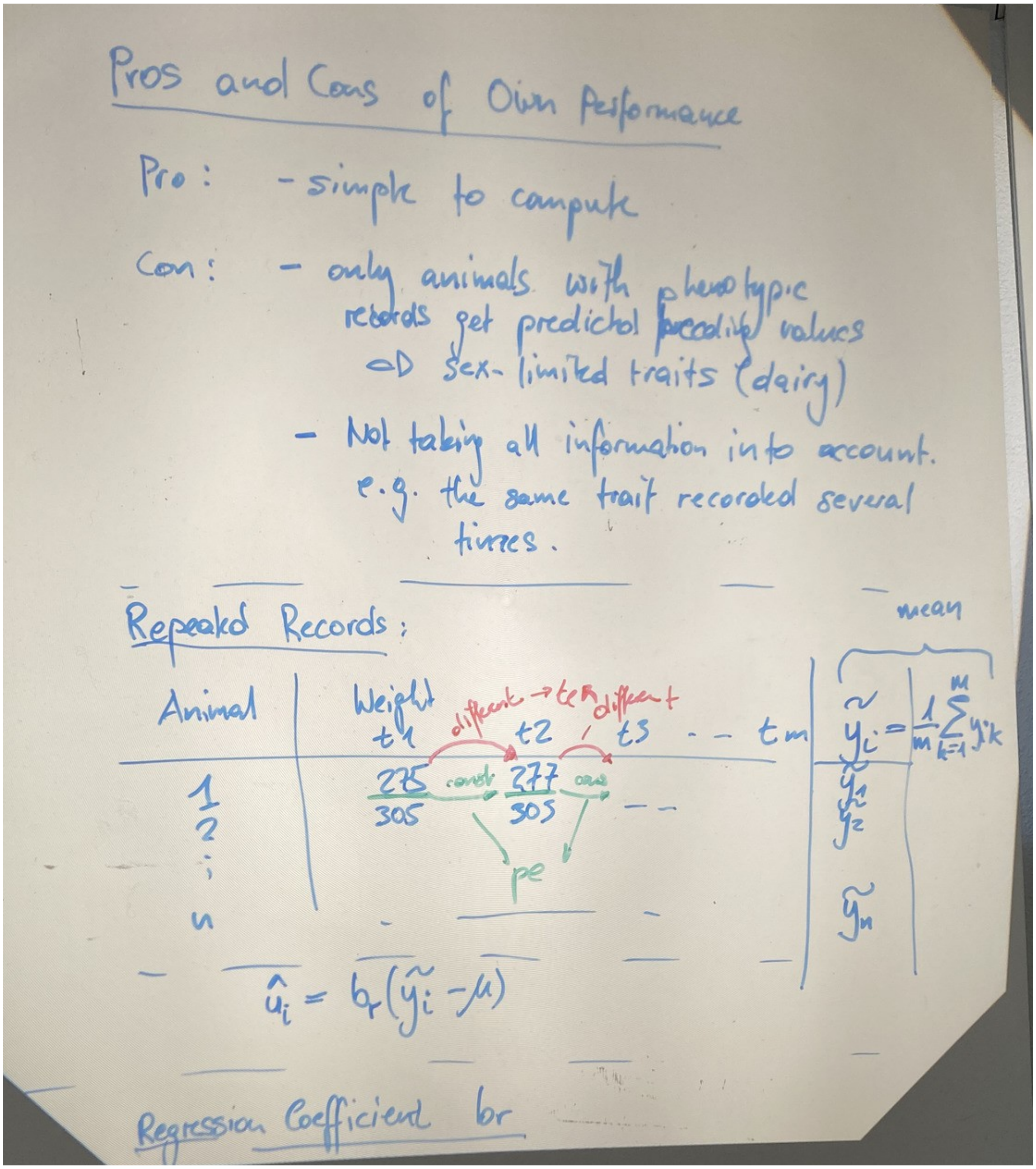
- Not taking all information into account. e.g. the same trait recorded several times.

Repeated Records:

Animal	Weight t_1	t_2	t_3	...	t_m	$\overbrace{\tilde{y}_i = \frac{1}{m}\sum_{k=1}^{m} y_{ik}}^{\text{mean}}$
1	275	277	--			$\tilde{y}_1$
2	305	305				$\tilde{y}_2$
$\vdots$						
n						$\tilde{y}_n$

$$\hat{u}_i = b_r(\tilde{y}_i - \mu)$$

OHP Picture 5



OHP Picture 6

Regression Coefficient b_r

$$b_r = \frac{Cov(u, \tilde{y})}{var(\tilde{y})}$$

- For $var(\tilde{y})$, we have an additional variance component:

$var(y)$ using for $y = \mu + u + e \rightarrow$ temp env (te)

$e \rightarrow$ permanent environment (pe

$$
\begin{aligned}
var(y) &= var(\mu + u + e) \\
&= var(\mu + u + pe + te) \\
&= \underbrace{var(\mu)}_{=\emptyset} + var(u) + var(pe) + var(te) \\
&= var(u) + var(pe) + var(te)
\end{aligned}
$$

- Define Repeatability t

$$t = \frac{var(u) + var(pe)}{var(y)} = \frac{\sigma_u^2 + \sigma_{pe}^2}{\sigma_y^2}$$

OHP Picture 7

Predicted Breeding Value $\hat{u}_i$:

$$\hat{u}_i = b_r(\tilde{y}_i - \mu)$$

$$b_r = \frac{cov(u,\tilde{y})}{var(\tilde{y})}$$

$$= \frac{cov(u_i, \mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij})}{var(\tilde{y})}$$

$$\tilde{y}_i = \frac{1}{m}\sum_{j=1}^{m} y_{j,i}$$

$$= \frac{1}{m}\sum_{j=1}^{m} \mu + u_i + pe_i + te_{ij}$$

$$= \frac{1}{m}\sum_{j=1}^{m}\mu + \frac{1}{m}\sum_{j=1}^{m} u_i + \frac{1}{m}\sum_{j=1}^{m} pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij}$$

$$= \boxed{\mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij}}$$

$$= \frac{\underbrace{cov(u_i,\mu)}_{=\emptyset} + cov(u_i,u_i) + \underbrace{cov(u_i,pe_i)}_{=\emptyset} + \underbrace{\frac{1}{m}\sum_{j=1}^{m} cov(u_i,te_{ij})}_{=\emptyset}}{var(\tilde{y})}$$

$$= \frac{cov(u_i,u_i)}{var(\tilde{y})} = \frac{var(u_i)}{var(\tilde{y})}$$

$$var(\tilde{y}) = var(\mu + u_i + pe_i + \frac{1}{m}\sum_{j=1}^{m} te_{ij})$$

$$= \underbrace{var(\mu)}_{\emptyset} + var(u_i) + var(pe_i) + \frac{1}{m^2}\sum_{j=1}^{m} \overbrace{var(te_{ij})}^{var(te_i)}$$

$$= var(u_i) + var(pe_i) + \frac{1}{m} var(te_i)$$

- Assumption: $cov(u,pe) = cov(u,te) = cov(te_{ij}, te_{i,k}) = 0$
- Use $t = \frac{\boxed{var(u) + var(pe)}}{var(y)} \rightarrow t \cdot var(y)$

OHP Picture 8

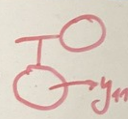
$$b_r = \frac{\mathrm{cov}(u,\tilde{y})}{\mathrm{var}(\tilde{y})} = \frac{\mathrm{var}(u)}{\left[t + \frac{1}{m}(1-t)\right]\mathrm{var}(y)} =$$

$$= \frac{m \cdot \mathrm{var}(u)}{\left[mt + (1-t)\right]\mathrm{var}(y)} = \frac{m \cdot \mathrm{var}(u)}{\left[(m-1)t + 1\right]\mathrm{var}(y)}$$

$$= \frac{m \cdot h^2}{1 + (m-1)t}$$

$$\Rightarrow \hat{u}_i = \boxed{\frac{m\, h^2}{1 + (m-1)t}} (\tilde{y}_i - \mu)$$

Progeny Records : Dairy Cattle $\hat{u}_1$ $\boxed{1}$



Animal	offspring 1	2	3	...	n	$\bar{y}_i$
1	$\boxed{y_{11}}$	$\boxed{y_{12}}$	-	-	y_{1n}	$\bar{y}_1 = \frac{1}{n}\sum_{k=1}^{n} y_{1k}$
2						$\bar{y}_2$
⋮						⋮
N						$\bar{y}_N$

OHP Picture 9

N

$\bar{y}_2$

$\vdots$

$\bar{y}_N$

Goal: Predict $\hat{u}_i$ based on $\bar{y}_i$

$$\hat{u}_i = b(\bar{y}_i - \mu)$$

$$b = \frac{cov(u, \bar{y}_\cdot)}{Var(\bar{y})}$$

- What is $\bar{y}_i$? $\bar{y}_i = \frac{1}{n}\sum_{k=1}^{n} y_{ik}$
- Use $y_{ik} = \mu + u_k + e_k$

$$= \mu + \underbrace{\frac{1}{2}u_i + \frac{1}{2}u_{dk} + m_k}_{u_k} + e_k$$

$$\bar{y}_i = \frac{1}{n}\sum_{k=1}^{n}\left(\mu + \frac{1}{2}u_i + \frac{1}{2}u_{dk} + m_k + e_k\right)$$

$$\circledast\ \mu + \frac{1}{2}u_i + \frac{1}{2n}\sum_{k=1}^{n} u_{dk} + \frac{1}{n}\sum_{k=1}^{n} m_k + \frac{1}{n}\sum_{k=1}^{n} e_k$$

$$cov(u_i, \bar{y}_i) = cov\left(u_i, \left[\mu + \frac{1}{2}u_i + \frac{1}{2n}\sum_{k=1}^{n} u_{dk} + \frac{1}{n}\sum_{k=1}^{n} m_k + \frac{1}{n}\sum_{k=1}^{n} e_k\right]\right)$$

OHP Picture 10

$$\operatorname{cov}(u_i,\bar{y}_i) = \operatorname{cov}\Big(u_i, \Big[\mu + \tfrac{1}{2}u_i + \tfrac{1}{2n}\sum_{k=1}^{n} u_{dk} + \tfrac{1}{n}\sum_{k=1}^{n} m_k + \tfrac{1}{n}\sum_{k=1}^{n} e_k\Big]\Big)$$

$$= \underbrace{\operatorname{cov}(u_i,\mu)}_{=0} + \operatorname{cov}(u_i,\tfrac{1}{2}u_i) + \underbrace{\operatorname{cov}\Big(u_i,\tfrac{1}{2n}\sum_{k=1}^{n} u_{dk}\Big)}_{=0} + \underbrace{\operatorname{cov}\Big(u_i,\tfrac{1}{n}\sum_{k=1}^{n} m_k\Big)}_{=0} + \underbrace{\operatorname{cov}\Big(u_i,\tfrac{1}{n}\sum_{k=1}^{n} e_k\Big)}_{=0}$$

$$= \operatorname{cov}(u_i,\tfrac{1}{2}u_i) = \tfrac{1}{2}\operatorname{cov}(u_i,u_i) = \tfrac{1}{2}\operatorname{var}(u)$$

(*)

$$\operatorname{var}(\bar{y}_i) = \operatorname{var}\Big(\mu + \tfrac{1}{2}u_i + \tfrac{1}{2n}\sum_{k=1}^{n} u_{dk} + \tfrac{1}{n}\sum_{k=1}^{n} m_k + \tfrac{1}{n}\sum_{k=1}^{n} e_k\Big)$$

$$= \underbrace{\operatorname{var}(\mu)}_{=0} + \operatorname{var}(\tfrac{1}{2}u_i) + \operatorname{var}\Big(\tfrac{1}{2n}\sum_{k=1}^{n} u_{dk}\Big) + \operatorname{var}\Big(\tfrac{1}{n}\sum_{k=1}^{n} m_k\Big) + \operatorname{var}\Big(\tfrac{1}{n}\sum_{k=1}^{n} e_k\Big)$$

$$= \boxed{\tfrac{1}{4}\operatorname{var}(u_i)} + \operatorname{var}\Big(\tfrac{1}{2n}\sum_{k=1}^{n} u_{dk}\Big) + \operatorname{var}\Big(\tfrac{1}{n}\sum_{k=1}^{n} m_k\Big) + \operatorname{var}\Big(\tfrac{1}{n}\sum_{k=1}^{n} e_k\Big)$$

permanent

$$= t\cdot\operatorname{var}(y) + \tfrac{1}{n}(1-t)\cdot\operatorname{var}(y) = \Big[t + (1-t)/n\Big]\operatorname{var}(y)$$

$$t = \frac{1/4\,\operatorname{var}(u_i)}{\operatorname{var}(y)} = \frac{h^2}{4}$$

$$1/2\cdot h^2\cdot\operatorname{var}(y)$$

OHP Picture 11

$$\ldots \; var(y)$$

$$b = \frac{Cov(u_i, \bar{y}_i)}{Var(\bar{y}_i)} = \frac{\overbrace{1/2 \cdot var(u_i)}^{1/2 \cdot h^2 \cdot var(y)}}{\left[t + (1-t)/n\right] \cdot var(y)}$$

$$= \frac{1/2\, h^2 \cdot \cancel{var(y)}}{\left[h^2/4 + (1 - h^2/4)/n\right] \cdot \cancel{var(y)}}$$

$$= \frac{1/2\, h^2}{h^2/4 + (1 - h^2/4)/n}$$

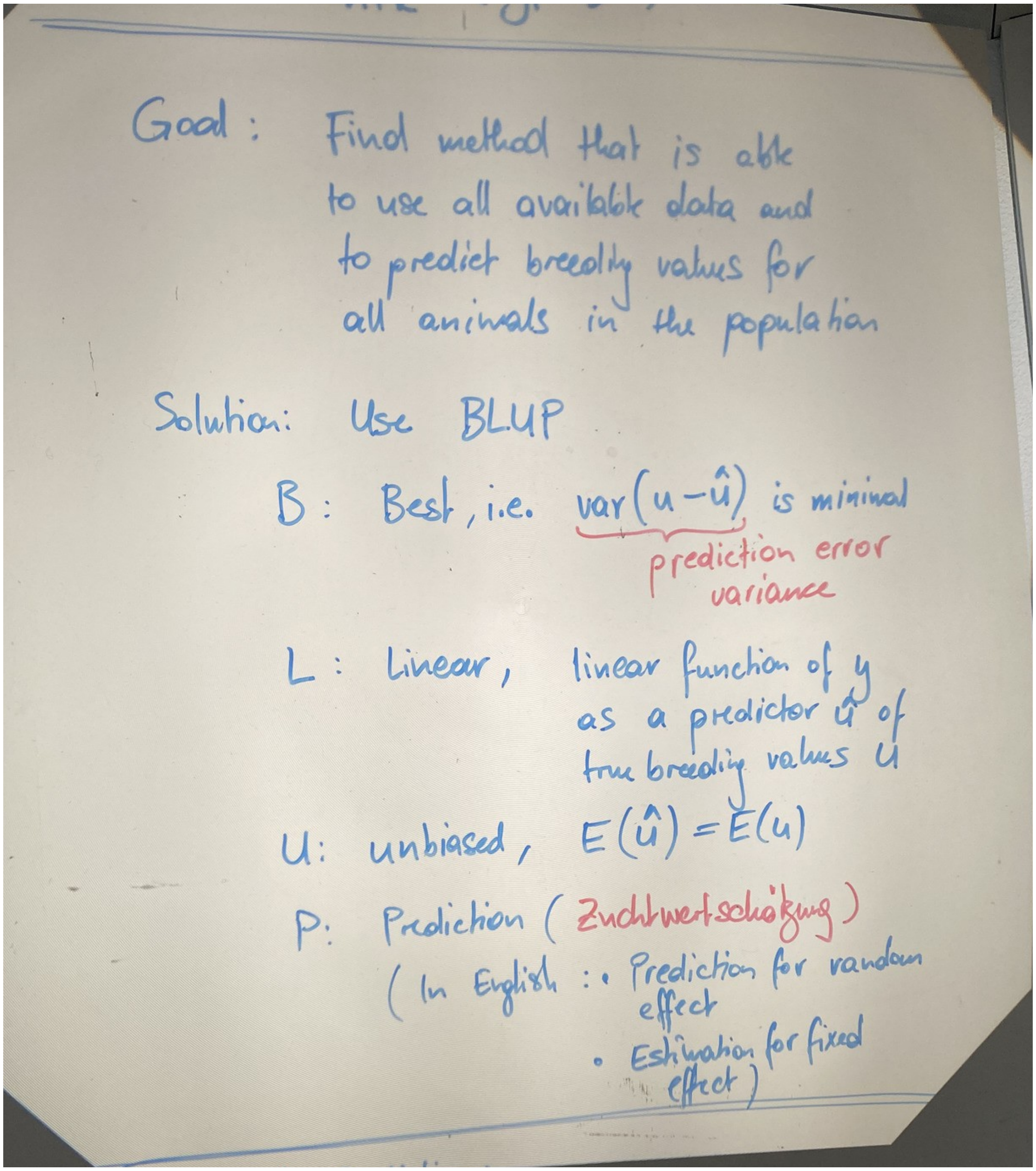
$$= \frac{1/2\, n h^2}{n h^2/4 + (1 - h^2/4)} = \frac{2 n h^2}{n h^2 + (4 - h^2)}$$

$$= \frac{2n}{n + (4 - h^2)/h^2} = \underline{\frac{2n}{n + k}}$$

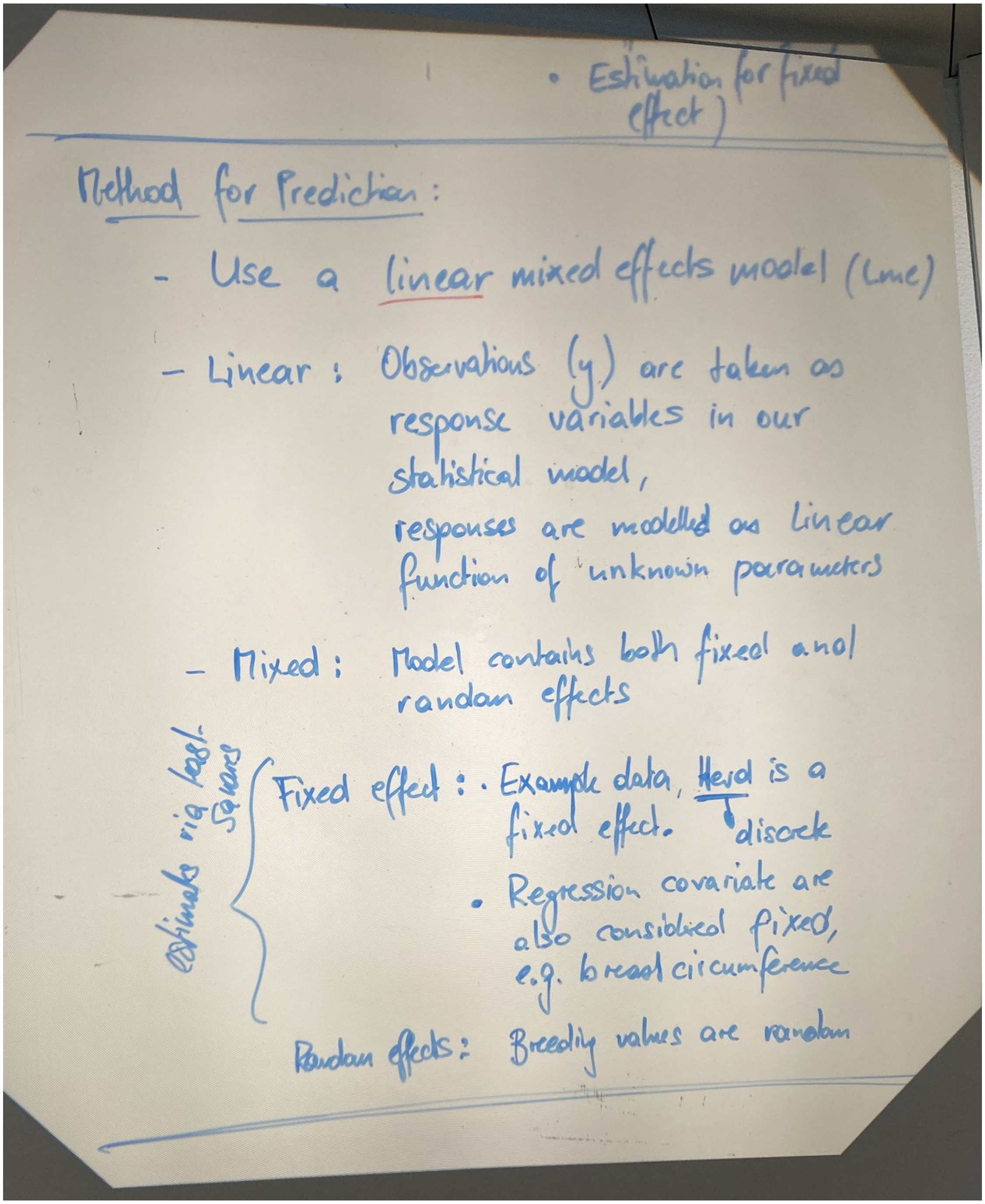
$$k = \frac{4 - h^2}{h^2}$$

$$\hat{u}_i = \frac{2n}{n+k}\left(\bar{y}_i - \mu\right)$$

OHP Picture 12



OHP Picture 13



OHP Picture 14