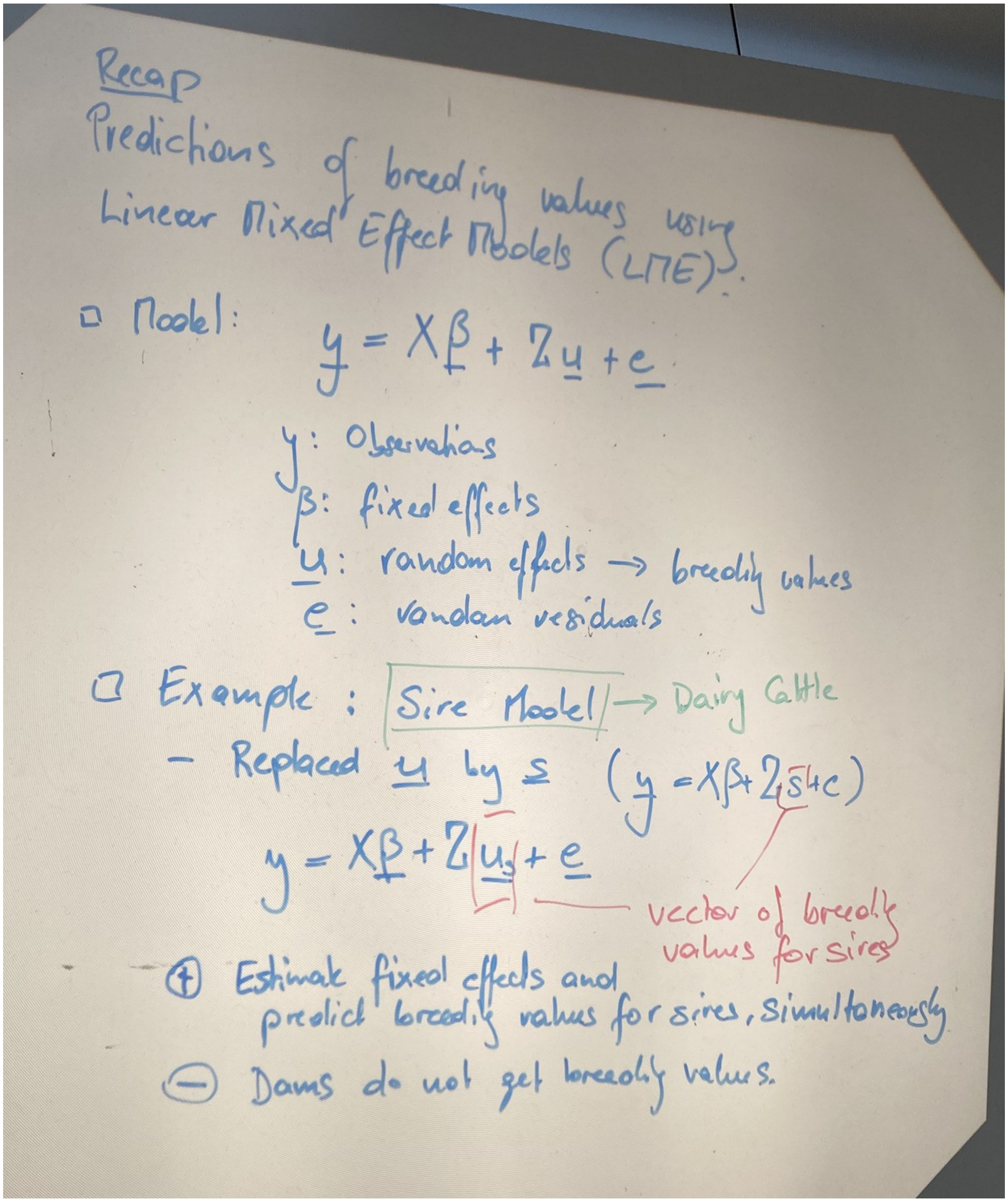
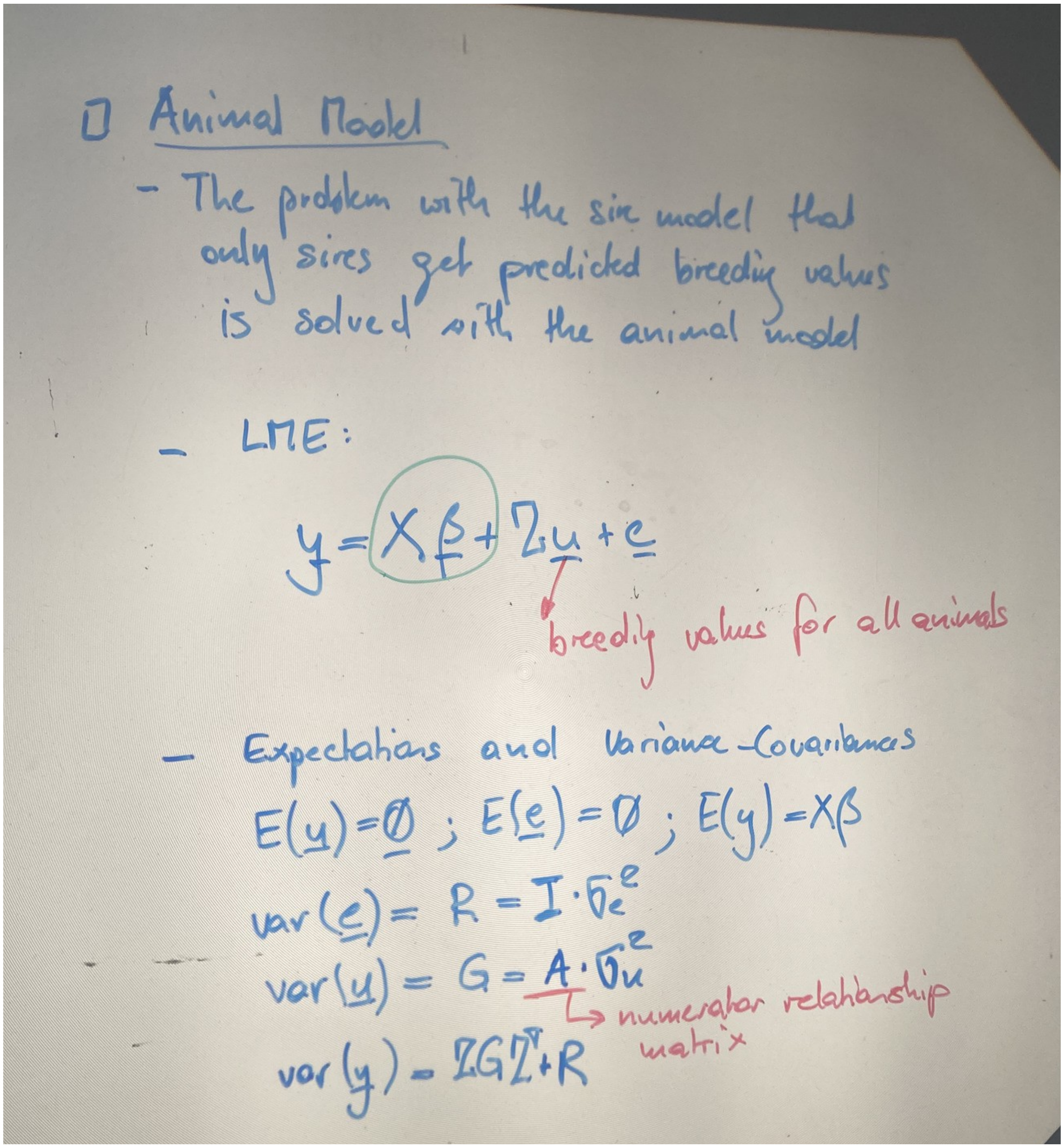
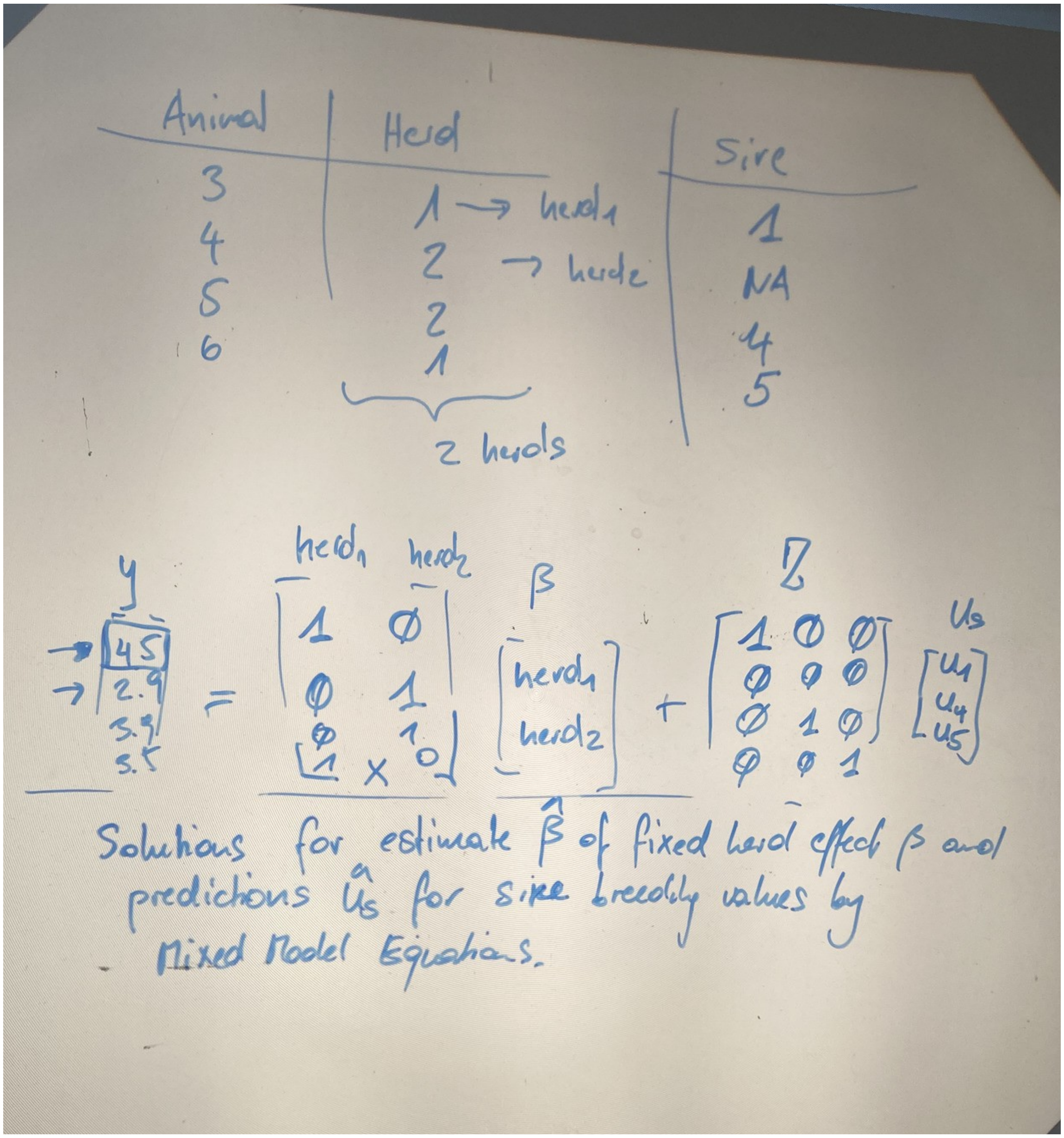
OHP Picture 1



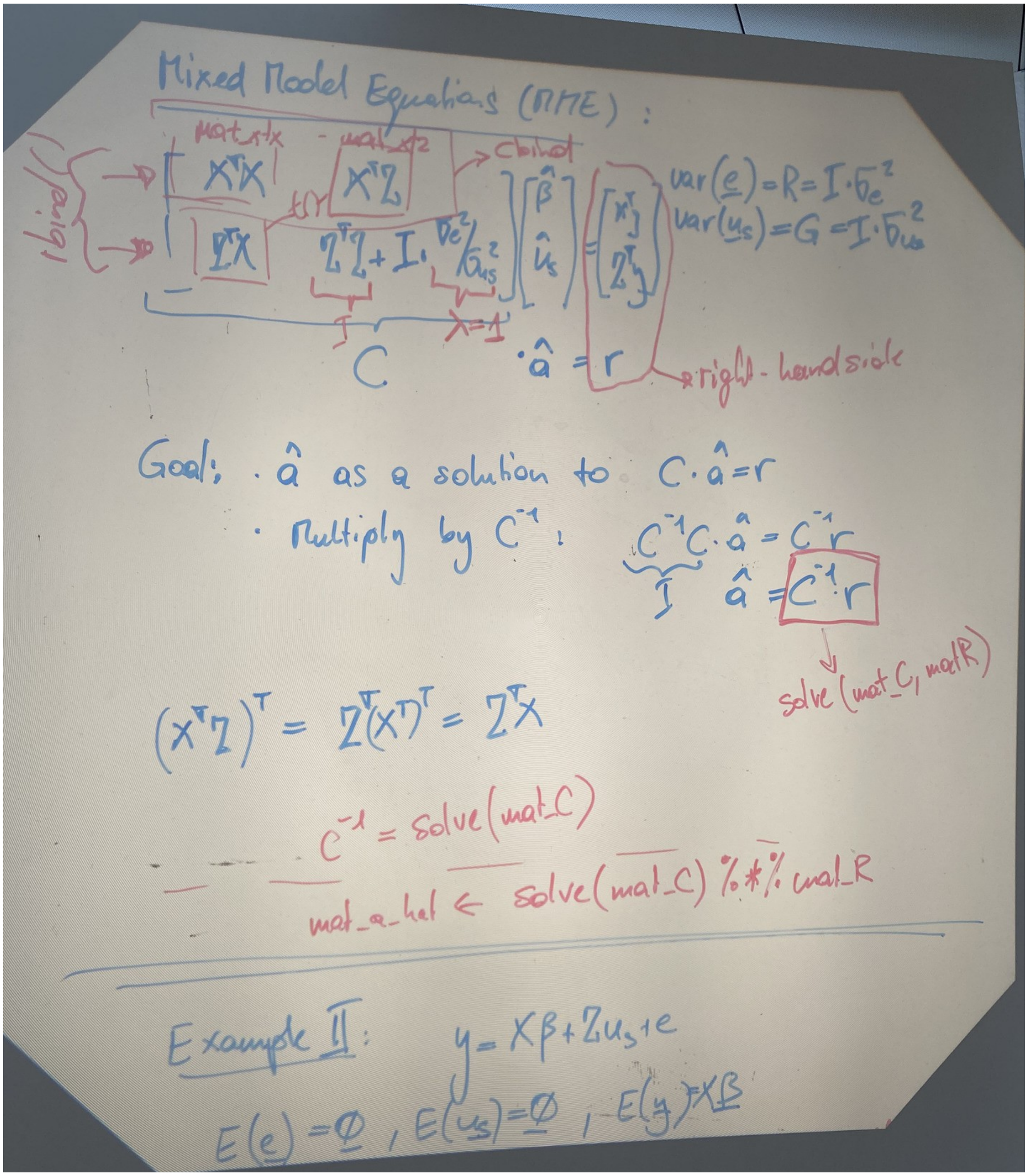
OHP Picture 2



OHP Picture 3



OHP Picture 4



Mixed Model Equations (MME):

rbind

mat_xtx, mat_xtz, cbind, t()

$$\underbrace{\begin{bmatrix} X^TX & \boxed{X^TZ} \\ \boxed{Z^TX} & \underbrace{Z^TZ}_{I} + I\cdot \underbrace{\frac{\sigma_e^2}{\sigma_{u_s}^2}}_{\lambda=1} \end{bmatrix}}_{C} \cdot \underbrace{\begin{bmatrix}\hat\beta \\ \hat u_s\end{bmatrix}}_{\hat a} = \underbrace{\begin{bmatrix} X^Ty \\ Z^Ty \end{bmatrix}}_{r}$$

right-hand side

$var(\underline{e}) = R = I\cdot\sigma_e^2$

$var(\underline{u_s}) = G = I\cdot\sigma_{u_s}^2$

Goal: · $\hat a$ as a solution to $C\cdot\hat a = r$

· Multiply by C^{-1}: $\underbrace{C^{-1}C}_{I}\cdot\hat a = C^{-1}r$

$\hat a = \boxed{C^{-1}\cdot r}$

solve(mat_C, matR)

$(X^TZ)^T = Z^T(X^T)^T = Z^TX$

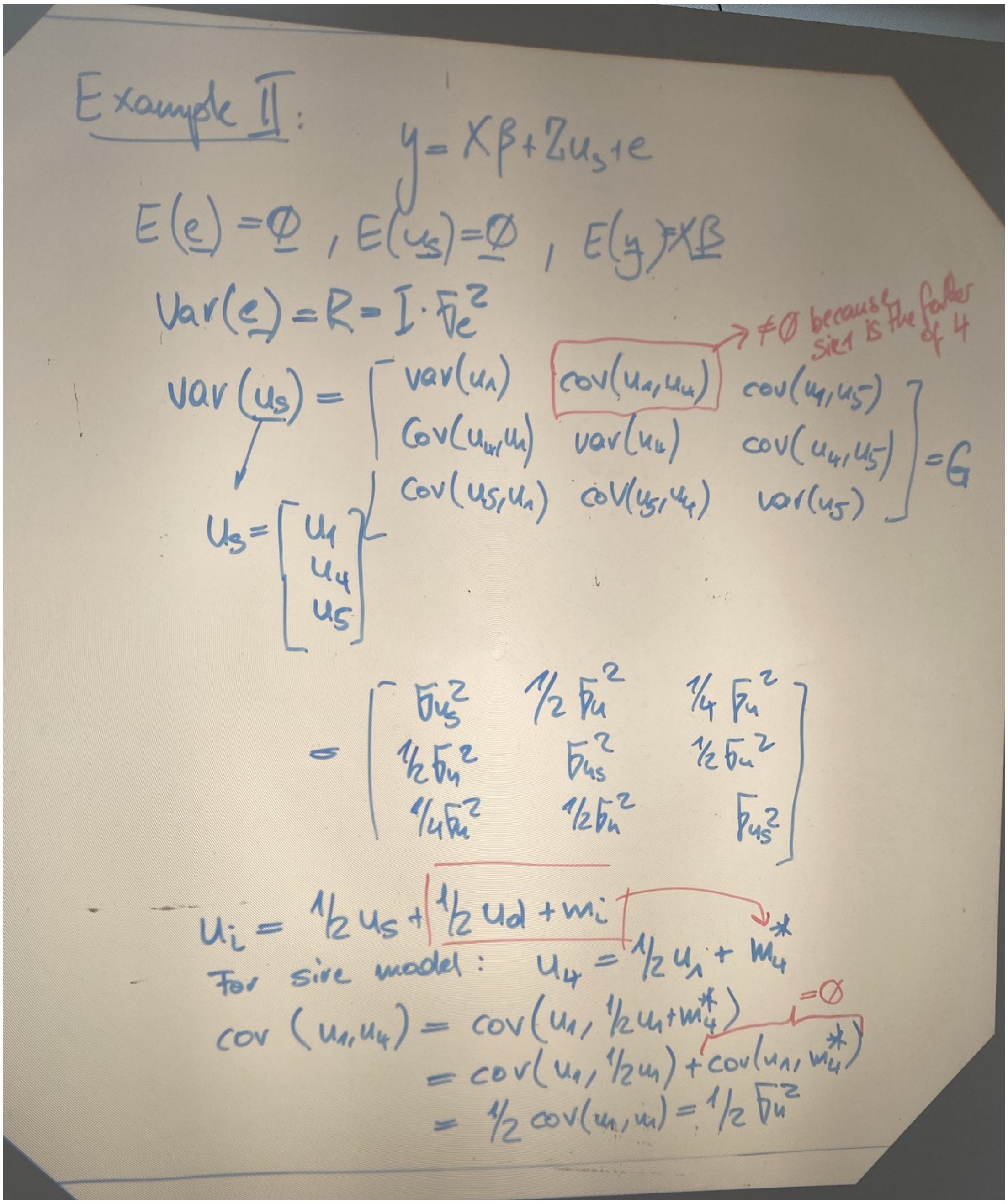
C^{-1} = solve(mat_C)

mat_a_hat <- solve(mat_C) %*% mat_R

Example II: $y = X\beta + Zu_s + e$

$E(\underline{e}) = \underline{0}$, $E(u_s) = \underline{0}$, $E(y) = X\beta$

OHP Picture 5



Example II:

$$y = X\beta + Zu_s + e$$

$$E(\underline{e}) = \underline{0}, \quad E(\underline{u_s}) = \underline{0}, \quad E(y) = X\underline{\beta}$$

$$Var(\underline{e}) = R = I \cdot \sigma_e^2$$

$$var(\underline{u_s}) = \begin{bmatrix} var(u_1) & \boxed{cov(u_1,u_4)} & cov(u_1,u_5) \\ cov(u_4,u_1) & var(u_4) & cov(u_4,u_5) \\ cov(u_5,u_1) & cov(u_5,u_4) & var(u_5) \end{bmatrix} = G$$

$\neq 0$ because sire 1 is the father of 4

$$u_s = \begin{bmatrix} u_1 \\ u_4 \\ u_5 \end{bmatrix}$$

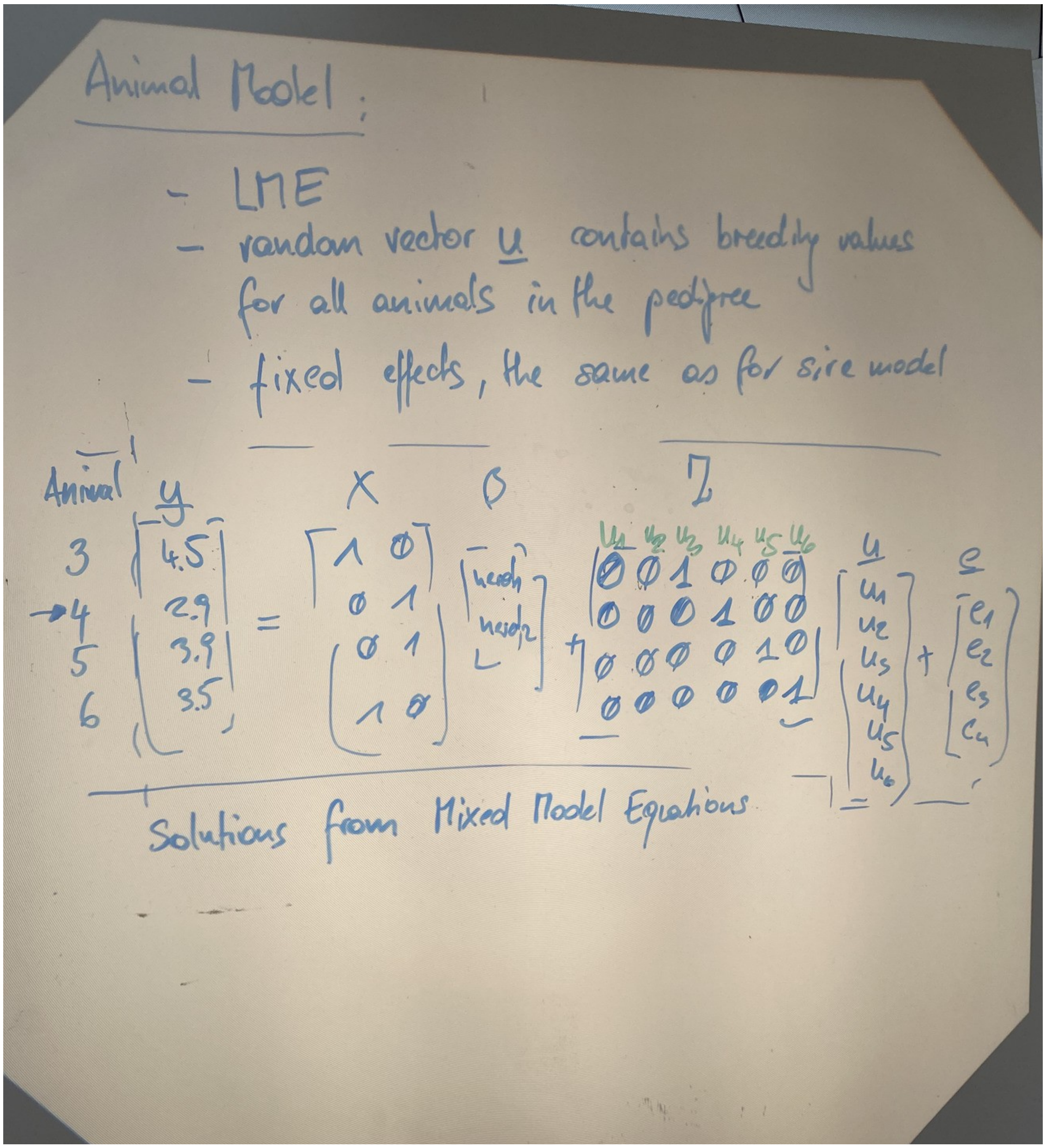
$$= \begin{bmatrix} \sigma_{us}^2 & 1/2\,\sigma_u^2 & 1/4\,\sigma_u^2 \\ 1/2\,\sigma_u^2 & \sigma_{us}^2 & 1/2\,\sigma_u^2 \\ 1/4\,\sigma_u^2 & 1/2\,\sigma_u^2 & \sigma_{us}^2 \end{bmatrix}$$

$$u_i = 1/2\,u_s + \underbrace{\boxed{1/2\,u_d + m_i}}_{m_4^*}$$

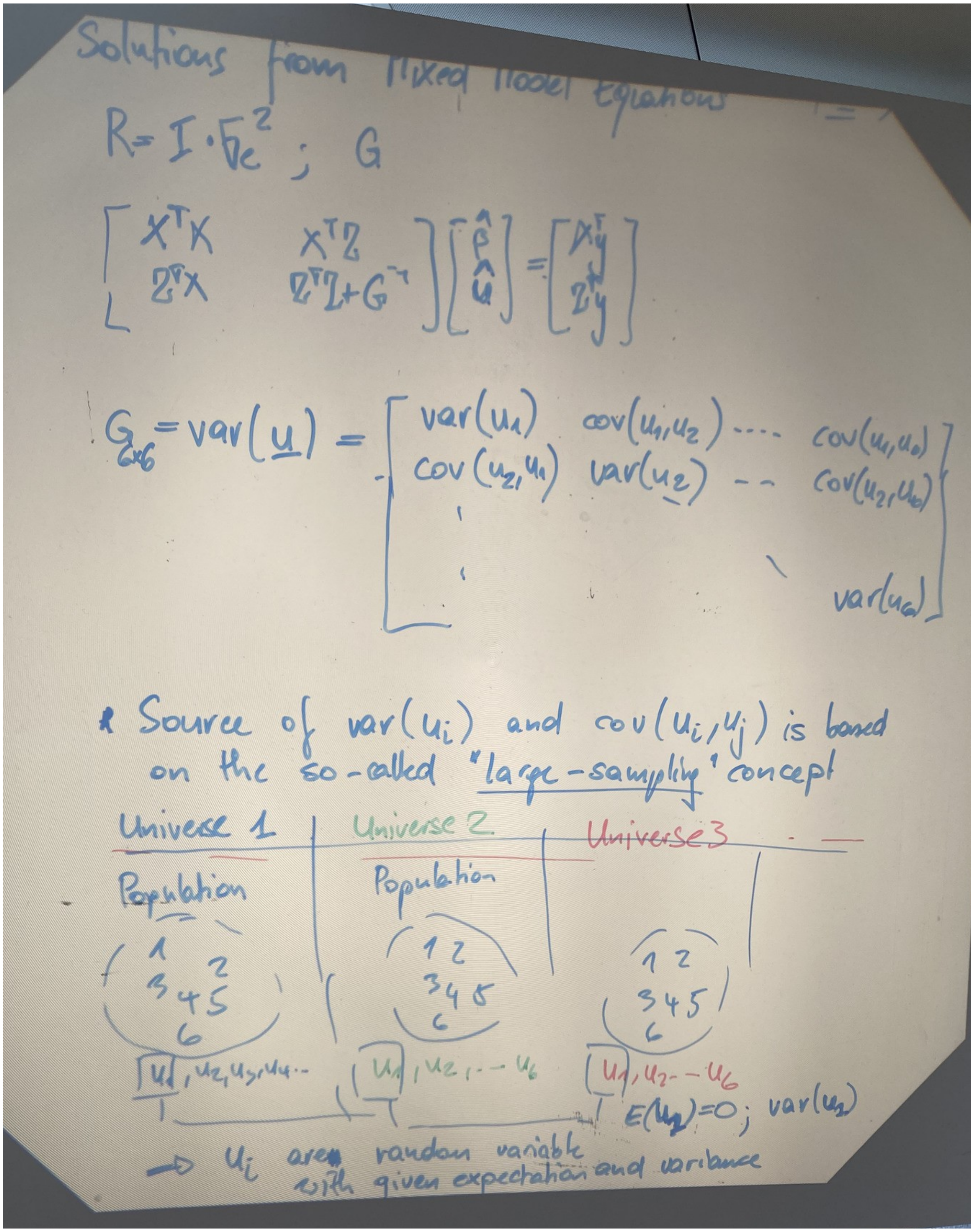
For sire model: $u_4 = 1/2\,u_1 + m_4^*$

$$cov(u_1,u_4) = cov(u_1, 1/2\,u_1 + m_4^*)$$
$$= cov(u_1, 1/2\,u_1) + \underbrace{cov(u_1, m_4^*)}_{=0}$$
$$= 1/2\,cov(u_1,u_1) = 1/2\,\sigma_u^2$$

OHP Picture 6



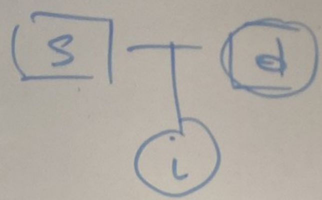
OHP Picture 7



OHP Picture 8

For animal i: $E(u_i) = \emptyset$

$$var(u_i) = (1 + F_i)\,\sigma_u^2$$



- Inbreeding coefficient of animal i
- $F_i = \frac{1}{2} A_{sd}$ where A_{sd} is the relationship between parents s and d

$$G = var(\underline{u})$$

$$= \begin{bmatrix} var(u_1) & cov(u_1, u_2) & cov(u_1, u_3) & cov(u_1, u_6) \\ cov(u_2, u_1 & var(u_2) & \text{—} & \end{bmatrix}$$

$$= \begin{bmatrix} (1+F_1)\sigma_u^2 & \emptyset & \frac{1}{2}\sigma_u^2 & \frac{1}{2}\sigma_u^2 \text{ —} \\ & (1+F_2)\cdot\sigma_u^2 & & \end{bmatrix}$$

$cov(u_1, u_2) = \emptyset$, because animals 1 and 2 do not share common ancestors

$$cov(u_1, u_3) = cov(u_1, \tfrac{1}{2}u_1 + \tfrac{1}{2}u_2 + m_3)$$

$$= cov(u_1, \tfrac{1}{2}u_1) + cov(u_1, \tfrac{1}{2}u_2) + \underbrace{cov(u_1, m_3)}_{=\emptyset}$$

$$= \tfrac{1}{2}cov(u_1, u_1) + \tfrac{1}{2}\boxed{cov(u_1, u_2)}$$

$$= \tfrac{1}{2}\, var(u_1) = \tfrac{1}{2}\sigma_u^2$$

OHP Picture 9

$$G = \begin{bmatrix} 1+F_1 & 0 & 1/2 & 1/2 & 1/4 & \cdots \\ & & & & & \\ & & & & & \end{bmatrix} \cdot \sigma_u^2$$

$A \rightarrow$ numerator relationship matrix

$G = A \cdot \sigma_u^2$ used in animal model

For Mixed Model Equations, we need G^{-1}

$$G^{-1} = (A \cdot \sigma_u^2)^{-1} = A^{-1} \cdot \sigma_u^{-2}$$

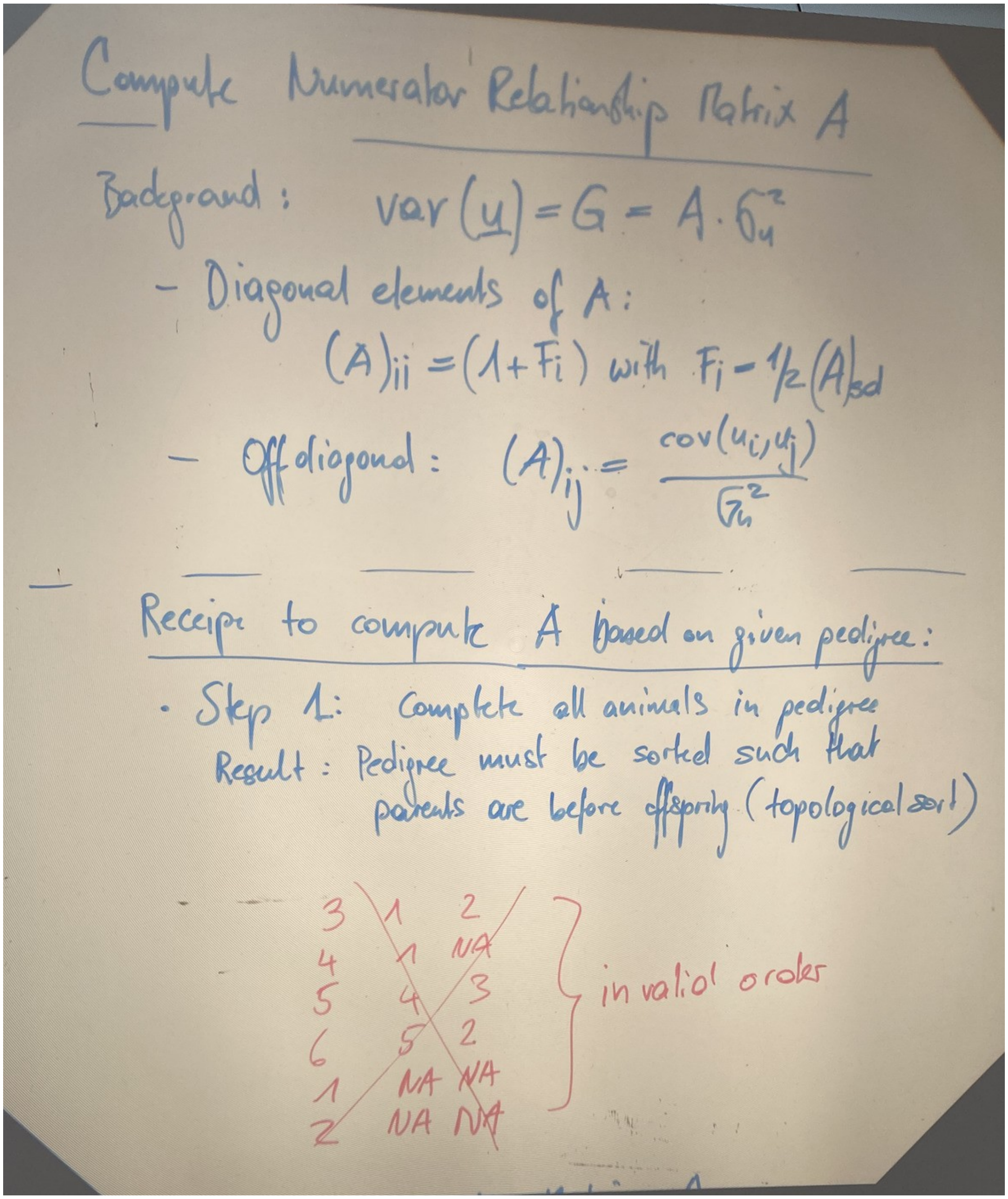
$$\begin{bmatrix} X^TX & X^TZ \\ Z^TX & Z^TZ + A^{-1} \cdot \lambda \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X^Ty \\ Z^Ty \end{bmatrix}$$

$$\lambda = \sigma_e^2 / \sigma_u^2$$

Compute Numerator Relationship Matrix A

Background: $var(\underline{u}) = G = A \cdot \sigma_u^2$

OHP Picture 10



OHP Picture 11

- Step 2: · Empty Matrix A: square
  - Dimension is the number of animals in pedigree
  - $$A_{6 \times 6} = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \boxed{1} & \boxed{0} & \frac{1}{2} & \frac{1}{2} & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{array}$$

    $(A)_{11}$ $(A)_{12}$

- Step 3: Diagonal element $(A)_{11} = 1 + F_1 = 1$
  Because animal 1 has unknown parents
  $F_1 = \emptyset$

- Step 4: Off-diagonal:
  $$(A)_{12} = 1/2 \left[\underbrace{(A)_{1,NA}}_{=\emptyset} + \underbrace{(A)_{1,NA}}_{=\emptyset} \right] = \emptyset$$
  $$(A)_{1\overset{1\ 2}{\vee}3} = 1/2 \left[(A)_{1,1} + (A)_{1,2} \right]$$
  $$= 1/2 \left[1 + \emptyset \right] = 1/2$$
  $$(A)_{1\overset{1\ NA}{\vee}4} = 1/2 \left[(A)_{1,1} + (A)_{1,NA} \right] = 1/2$$
  $$\vdots$$

Step 5: Copy first row into first column

OHP Picture 12