

Computing Elements of A

- Input always consists of a pedigree
- Skp 1: Complete pedigree, order such that parents are always before offspring
- Start with empty A. A is a square and symmetric matrix with number of rows equal to number of columns equal to the number of animals in the pedigree.

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 1 & 0 & 1/2 & 1/2 & 1/2 & 1/4 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1 & 0 \\ 1/4 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

Diagonal (Main-Dia)

$$(A)_{ij} = 1 + F_i$$

$$F_i = 0.5 \cdot (A)_{sd}$$

$$(A)_{11} = 1 + F_1 = 1 + 0 = 1$$

Off-Diag :

$$(A)_{hi} = \frac{1}{2} [(A)_{hs} + (A)_{id}]$$

where s and d are parents of i

$$(A)_{12} = \frac{1}{2} [(A)_{11} + (A)_{12}] = 0$$

$$(A)_{13} = \frac{1}{2} [(A)_{11} + (A)_{12}] - \frac{1}{2} [1 + 0] = \frac{1}{2}$$

OHP Picture 3

$$(A)_{14} = \frac{1}{2} [(A)_{11} + (A)_{1M}] = \frac{1}{2} [1 + 0] = \frac{1}{2}$$

$$(A)_{15} = \frac{1}{2} [(A)_{14} + (A)_{13}] = \frac{1}{2} [\frac{1}{2} + \frac{1}{2}] = \frac{1}{2}$$

$$(A)_{16} = \frac{1}{2} [(A)_{15} + (A)_{12}] = \frac{1}{2} [\frac{1}{2} + 0] = \frac{1}{4}$$

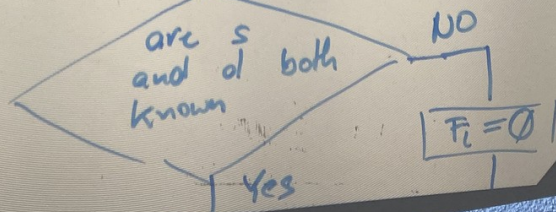
$$(A)_{22} = 1 + F_2 = 1 + \frac{1}{2} (A)_{1MNA} = 1$$

Computation of $(A)_{ii}$ requires F_i and $(A)_{sd}$

$$(A)_{ii} = 1 + F_i = 1 + 0.5 \cdot (A)_{sd}$$

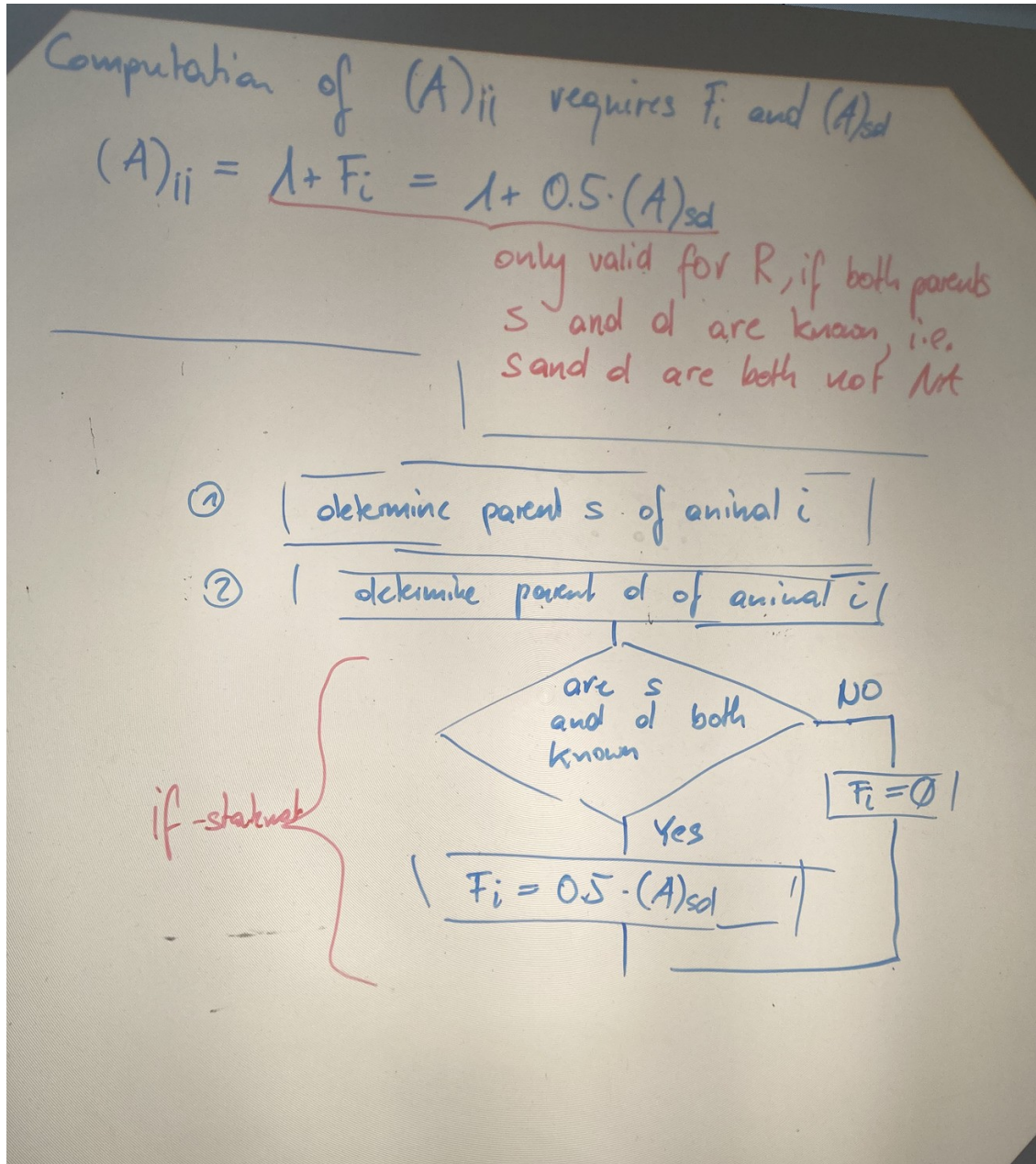
only valid for R, if both parents
s and d are known, i.e.
s and d are both not NA

- ① determine parent s of animal i
- ② determine parent d of animal i



if-statement

OHP Picture 4



For MRE:

□ Require A^{-1} not A where inverse A^{-1} is defined as the matrix, that satisfies

$$A \cdot A^{-1} = I$$

□ In R: use `pedigreemm::getA()` for A

- For A^{-1} `pedigreemm::getAinv()` for A^{-1}

□ MRE for large data sets ($10^6 - 10^7$ records) are only possible, because A^{-1} can directly be constructed from the pedigree without first computing A

↓
major selection response together with AI before genomic selection

Decomposing Breeding values

For animal i with parents s and o :

Decomposing Breeding values

□ For animal i with parents s and d :
 The breeding value u_i can be decomposed as:

$$u_i = \frac{1}{2} u_s + \frac{1}{2} u_d + m_i$$

□ For all animals in Pedigree:

$$\begin{cases} u_1 = m_1 \\ u_2 = m_2 \\ u_3 = m_3 \\ u_4 = \frac{1}{2} u_1 + \frac{1}{2} u_2 + m_4 \\ u_5 = \frac{1}{2} u_3 + \frac{1}{2} u_2 + m_5 \end{cases} \quad \text{Matrix-Vector notation}$$

$$\underline{u} = \begin{matrix} & \underline{p} & \\ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} & = & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{bmatrix} \end{matrix}$$

Decomposition of $\text{var}(u_i)$

$$\begin{aligned}\text{var}(u_i) &= \text{var}\left(\frac{1}{2}u_s + \frac{1}{2}u_d + m_i\right) \\ &= \text{var}\left(\frac{1}{2}u_s\right) + \text{var}\left(\frac{1}{2}u_d\right) + \text{var}(m_i) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_s, \frac{1}{2}u_d\right) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_s, m_i\right) \\ &\quad + 2\text{cov}\left(\frac{1}{2}u_d, m_i\right) \quad \left. \vphantom{\begin{aligned} &+ 2\text{cov}\left(\frac{1}{2}u_s, m_i\right) \\ &+ 2\text{cov}\left(\frac{1}{2}u_d, m_i\right) \end{aligned}} \right\} = 0 \\ &= \frac{1}{4}\text{var}(u_s) + \frac{1}{4}\text{var}(u_d) + \text{var}(m_i) \\ &\quad + \frac{1}{2}\text{cov}(u_s, u_d)\end{aligned}$$