3.5 Appendix: Parent-Offspring Breeding Values

In section 3.1.2 the decomposition of the breeding value u_i for animal i into half of the breeding values of parents s and d plus the mendelian sampling effect m_i is shown. The mendelian sampling effect m_i models the deviation of the offspring breeding value u_i from the parent average. Because m_i is modelled as a deviation, we can also state that over a large number (N) of offspring of parents s and d the average over all mendelian sampling effects is 0. This can be expressed with the following formula

$$\frac{1}{N}\sum_{i=1}^{N} m_i = 0 \tag{3.27}$$

where m_i are mendelian sampling effects of N offspring from the same parents s and d.

3.5.1 Single Locus Model

Let us return for a moment to the single locus model and try to illustrate whether the proposition in equation (3.27) is true. This requires to distinguish between different cases according to the genotypes of parents s and d at the locus G of interest.

3.5.1.1 Case 1: Homozygous Parents With Same Genotype

If both parents s and d are homozygous and have the same genotype either both G_1G_1 or both G_2G_2, each offspring has the same genotype as the parents. If parents and offspring have the same genotypes, then they have also the same breeding values and hence every m_i term is 0 and with that equation (3.27) is satisfied.

3.5.1.2 Case 2: Homozygous Parents With Different Genotypes

In this case one of the parents - let us say s has genotype G_1G_1 with breeding value $u_s = 2q\alpha$ and the other parent d has genotype G_2G_2 with breeding value $u_d = -2p\alpha$. All offspring form such a mating are heterozygous with breeding value $(q-p)\alpha$. Inserting this into the decomposition gives

$$u_i = \frac{1}{2}(u_s + u_d) + m_i$$

Therefore

$$m_i = u_i - \frac{1}{2}(u_s + u_d) = (q-p)\alpha - \frac{1}{2}(2q\alpha - 2p\alpha) = 0$$

This shows that also in this case every individual m_i term is 0 and hence the average m_i effect over a large number of offspring from the same parents is also 0.

3.5.1.3 Case 3: One Parent Homozygous, the Other Parent Heterozygous

One parent is homozygous either G_1G_1 or G_2G_2 and the other parent is heterozygous G_1G_2. Let us assume that parent s has genotype G_1G_1 with breeding value $u_s = 2q\alpha$ and parent d is heterozygous with breeding value $u_d = (q-p)\alpha$. The parent average breeding value is.

$$\frac{1}{2}(u_s + u_d) = \frac{1}{2}(2q\alpha + (q-p)\alpha) = \frac{3}{2}q\alpha - \frac{1}{2}p\alpha$$

Parents s and d can have either an offspring i with genotype G_1G_1 or an offspring j with genotype G_1G_2. Both types of offspring can be observed with probability 1/2. Using the above parent average, m_i and m_j can be computed as

$$m_i = u_i - \frac{1}{2}(u_s + u_d) = 2q\alpha - \frac{3}{2}q\alpha + \frac{1}{2}p\alpha = \frac{1}{2}\alpha$$

Similarly

$$m_j = u_j - \frac{1}{2}(u_s + u_d) = (q-p)\alpha - \frac{3}{2}q\alpha + \frac{1}{2}p\alpha = -\frac{1}{2}\alpha$$

Taking the average over a large number of offspring where m_i and m_j can be observed with probability 1/2 leads to

$$\frac{1}{N}\sum_{k=1}^{N} m_k = \frac{1}{2}(\frac{1}{2}\alpha - \frac{1}{2}\alpha) = 0$$

The case where parent s has genotype G_2G_2 and parent d has genotype G_1G_2 is left to the reader as an exercise.

3.5.1.4 Case 4: Heterozygous Parents

Heterozygous parents have heterozygous offspring with probability 1/2 and they have homozygous G_1G_1 offspring and homozygous G_2G_2 offspring both with probability 1/4. The parent average breeding value for heterozygous parents is

$$\frac{1}{2}(u_s + u_d) = (q - p)\alpha$$

For all heterozygous offspring of parents s and d, each individual m_i term is equal to 0. Based on the concept of allele substitution, we know that the difference between the breeding value of the G_1G_1 offspring and the parent average is α. Furthermore the difference between the breeding value of the G_2G_2 offspring and the parent average is $-\alpha$. Because the homozygous offspring occur with the same probability, the average mendelian sampling effect for a large number of offspring for heterozygous parents is 0. The detailed computations are left to the reader as an exercise.