Livestock Breeding and Genomics - Exercise 10

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Problem 1: Inverse Numerator Relationship Matrix

During the lecture the method of computing the inverse numerator relationship matrix A^{-1} directly was introduced. The computation is based on the LDL-decomposition. As a result, we can write

$$A^{-1} = (L^T)^{-1} \cdot D^{-1} \cdot L^{-1}$$

where $L^{-1} = I - P$, and D^{-1} is a diagonal matrix with $(D^{-1})_{ii} * \sigma_u^{-2} = var(m_i)^{-1}$.

Tasks

- Use the example pedigree given below and compute the matrices L^{-1} and D^{-1} to compute A^{-1}
- Verify your result using the function `getAinv()` from package pedigreemm.

Pedigree

```
nr_animal <- 6
tbl_pedigree <- tibble::tibble(Calf = c(1:nr_animal),
                               Sire = c(NA, NA, NA, 1 ,3, 4),
                               Dam = c(NA, NA, NA, 2, 2, 5))
tbl_pedigree
```

```
## # A tibble: 6 x 3
##    Calf  Sire   Dam
##   <int> <dbl> <dbl>
## 1     1    NA    NA
## 2     2    NA    NA
## 3     3    NA    NA
## 4     4     1     2
## 5     5     3     2
## 6     6     4     5
```

Problem 2: Rules

The following diagram helps to illustrate the rules for constructing A^{-1}

D^{-1}

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1	0	0	0	0	0
[2,]	0	1	0	0	0	0
[3,]	0	0	1	0	0	0
[4,]	0	0	0	2	0	0
[5,]	0	0	0	0	2	0
[6,]	0	0	0	0	0	2

L^{-1}

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1.0	0.0	0.0	0.0	0.0	0
[2,]	0.0	1.0	0.0	0.0	0.0	0
[3,]	0.0	0.0	1.0	0.0	0.0	0
[4,]	-0.5	-0.5	0.0	1.0	0.0	0
[5,]	0.0	-0.5	-0.5	0.0	1.0	0
[6,]	0.0	0.0	0.0	-0.5	-0.5	1

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1	0	0	-0.5	0.0	0.0
[2,]	0	1	0	-0.5	-0.5	0.0
[3,]	0	0	1	0.0	-0.5	0.0
[4,]	0	0	0	1.0	0.0	-0.5
[5,]	0	0	0	0.0	1.0	-0.5
[6,]	0	0	0	0.0	0.0	1.0

$(L^T)^{-1}$

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1	0	0	-1	0	0
[2,]	0	1	0	-1	-1	0
[3,]	0	0	1	0	-1	0
[4,]	0	0	0	2	0	-1
[5,]	0	0	0	0	2	-1
[6,]	0	0	0	0	0	2

$(L^T)^{-1} \cdot D^{-1}$

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
[1,]	1.5	0.5	0.0	-1.0	0.0	0
[2,]	0.5	2.0	0.5	-1.0	-1.0	0
[3,]	0.0	0.5	1.5	0.0	-1.0	0
[4,]	-1.0	-1.0	0.0	2.5	0.5	-1
[5,]	0.0	-1.0	-1.0	0.5	2.5	-1
[6,]	0.0	0.0	0.0	-1.0	-1.0	2

A^{-1}

Tasks

- Go through the list of animals in the pedigree and write down the contributions that are made to the different elements of matrix A^{-1}
- Based on the different contributions, try to come up with some general rules

Problem 3: Program using the Rules

Write a program in R that implements the rules found in the solution of Problem 2. Test your program with the pedigree given in Problem 1. Compare the results that you obtain with the result obtained from the function `pedigreemm::getAinv()`.