

OHP Picture 1

Recep 2023-10-06 ①

□ Breeding Value for animal with genotype $G_i G_j$ ($G_1 G_1, G_1 G_2, G_2 G_2$) assuming that locus G has an effect on quantitative trait

$\Rightarrow BV_{ij} = 2(\mu_{ij} - \mu)$ where μ_{ij} : mean genotypic value of a large number of offspring of animal with genotype $G_i G_j$

• μ : population mean

□ Population mean (μ)

$\mu = E[V] = (p-q)a + 2pqd$ where $f(G_1) = p$
 $f(G_2) = q$

$V_{11} = a; V_{12} = d; V_{22} = -a$

OHP Picture 2

Parent S with genotype: G_1G_1 (2)

Mate of S (random sample of population)
 $\Rightarrow f(G_1) = p$
 $f(G_2) = q$

Alleles of S	G_1 with $f(G_1) = p$	G_2 with $f(G_2) = q$
G_1 (with $f(G_1) = p$)	$f(G_1G_1) = 1 \cdot p = p$	$f(G_1G_2) = 1 \cdot q = q$
G_2 (with $f(G_2) = q$)	0	0

$$\mu_m = f(G_1G_1) \cdot V_m + f(G_1G_2) \cdot V_{12}$$

$$\mu_m = f(G_1G_1) \cdot a + f(G_1G_2) \cdot d$$

$$= p \cdot a + q \cdot d$$

$$BV_m = 2(\mu_m - \mu)$$

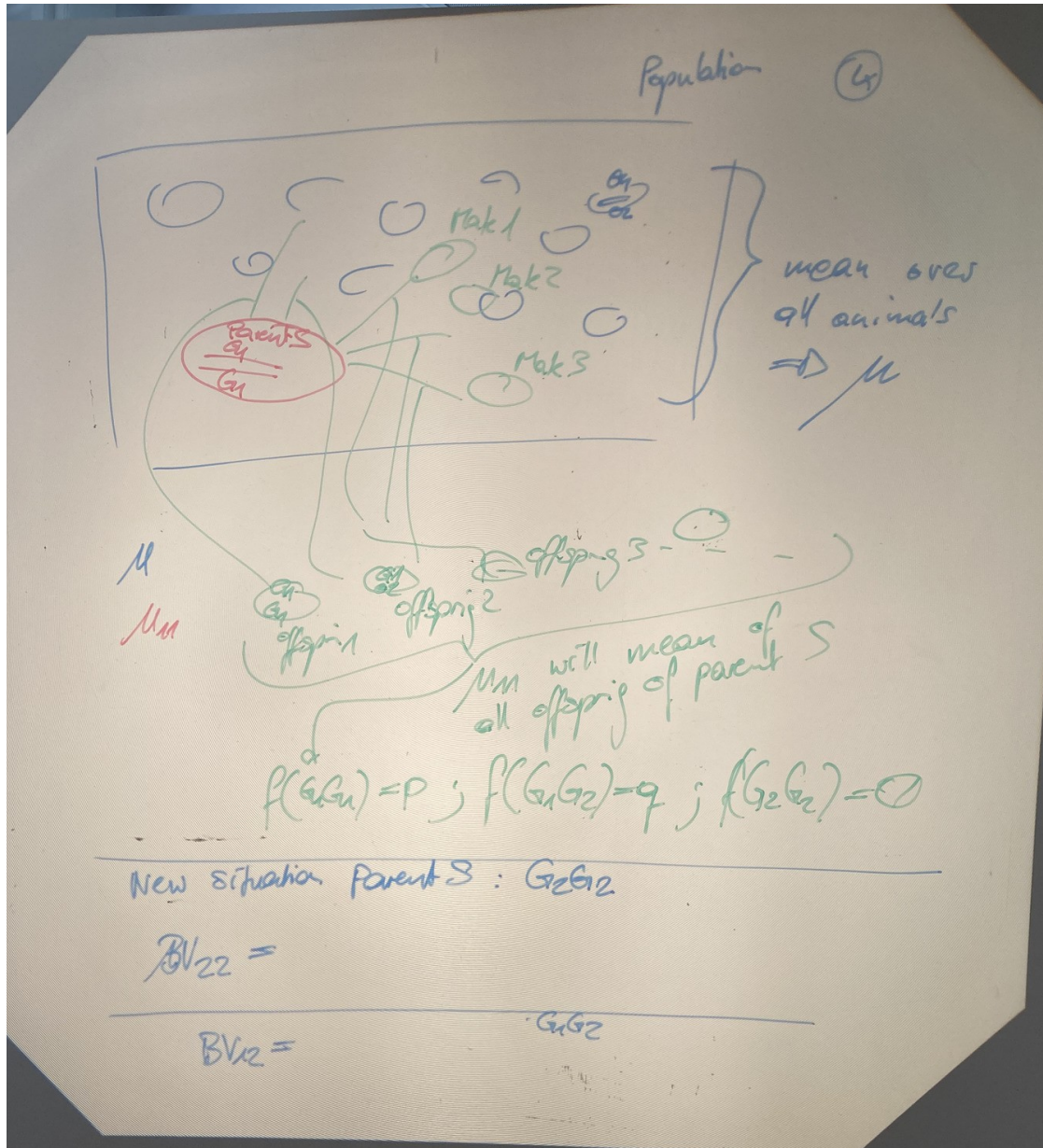
$$= 2(pa + qd - [(p+q)a + 2pqd])$$

OHP Picture 3

Breeding value BV_M for parent S with genotype G_1G_1 ③

$$\begin{aligned}
 BV_M &= 2(p_a + q_d - [(p-q)a + 2pqd]) \\
 &= 2(p_a + q_d - (p-q)a - 2pqd) \\
 &= 2(\cancel{p_a} + q_d - \cancel{p_a} + q_a - 2pqd) \\
 &= 2(q_d + q_a - 2pqd) \\
 &= 2(\underline{q_a} + \underline{q_d} - 2pqd) \\
 &= 2q(a + \underline{d} - 2pd) \\
 &= 2q(a + (1 - 2p)d) \quad ; \quad 1 = p + q \\
 &= 2q(a + (p + q - 2p)d) \\
 &= 2q(\underline{a + (q - p)d}) = 2q\alpha \\
 &\quad \quad \quad = \alpha
 \end{aligned}$$

OHP Picture 4



OHP Picture 5

Breeding Value for parent S with genotype G_2G_2 (5)

G_2G_2 :

$$BV_{z_2} = 2(\mu_{z_2} - \mu)$$

$$\mu_{z_2} = \overbrace{pd}^{G_1G_2} - \overbrace{qa}^{G_2G_1}$$

$$BV_{z_2} = 2(pd - qa - [(p-q)a + 2pqd])$$
~~$$= 2(-pa + pd)$$~~

$$= 2(pd - qa - (p-q)a - 2pqd)$$

$$= 2(pd - \cancel{qa} - pa + \cancel{qa} - 2pqd)$$

$$= 2(pd - pa - 2pqd)$$

$$= 2(-pa + pd - 2pqd)$$

$$= 2(-pa + (p - 2pq)d)$$

$$= 2(-pa + p(1 - 2q)d)$$

$$= 2p(-a + (1 - 2q)d)$$

$$= -2pa - (1 - 2q)d$$

$$= -2p(a + (q - p)d) = -2p\alpha$$

$p + q - 2q$
 $= p - q$

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G₁G₂
offspring : $f(G_1G_2) = \frac{1}{2}p + \frac{1}{2}q$
 $= \frac{1}{2}(p+q) = \frac{1}{2}$
 $= 1$

Parents	Males G ₁ f(G ₁)p	G ₂ with f(G ₂)=q	f(G ₁ G ₂)
G ₁ with f(G ₁)=0.5	f(G ₁ G ₂)=1/2 p	f(G ₁ G ₂)=1/2 q	f(G ₁ G ₂) = 0.5p
G ₂ · f(G ₂)=0.5	f(G ₁ G ₂)=1/2 p	f(G ₂ G ₂)=1/2 q	f(G ₁ G ₂) = 1/2(p+q) = 1/2

$$M_{12} = 0.5p \cdot a + 0.5d + 0.5q \cdot a$$

$$= \frac{1}{2}(pa + d + qa)$$

$$= \frac{1}{2}[(p+q)a + d]$$

$$BV_{12} = 2(0.5[(p+q)a + d]) - [(p+q)a + 2pd]$$

$$= (q-p)[a + (q-p)d] = (q-p)\alpha$$

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Summary: Breeding Values BV_{ij}

Genotype	Breeding Value
$G_1 G_1$	$2q\alpha$
$G_1 G_2$	$2q\alpha$ $(q-p)\alpha$
$G_2 G_2$	$-2p\alpha$

$\left. \begin{array}{l} 2q\alpha \\ (q-p)\alpha \\ -2p\alpha \end{array} \right\} \begin{array}{l} \alpha = a + (q-p)d \\ \text{if } d=0 \\ \Rightarrow \alpha = a \end{array}$

- Breeding Values are population-specific (only valid for one population / one Breed) because they depend on allele frequencies
- Expected values

$$\begin{aligned}
 E[BV] &= f(G_1 G_1) \cdot BV_{11} + f(G_1 G_2) \cdot BV_{12} + f(G_2 G_2) \cdot BV_{22} \\
 &= p^2 \cdot 2q\alpha + 2pq \cdot (q-p)\alpha + q^2 \cdot (-2p\alpha) \\
 &= \underbrace{2p^2 q \alpha}_{\cancel{2p^2 q \alpha}} + \underbrace{2pq^2 \alpha}_{\cancel{2pq^2 \alpha}} - \underbrace{2p^3 q \alpha}_{\cancel{2p^3 q \alpha}} - \underbrace{2pq^2 \alpha}_{\cancel{2pq^2 \alpha}} \\
 &= 0
 \end{aligned}$$

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Pairwise Differences of BV

$$\begin{aligned}
 BV_{11} - BV_{12} &= 2q\alpha - (q-p)\alpha \\
 &= 2q\alpha - q\alpha + p\alpha \\
 &= q\alpha + p\alpha = (p+q)\alpha = \alpha
 \end{aligned}$$

$$\begin{aligned}
 BV_{12} - BV_{22} &= (q-p)\alpha - (-2p\alpha) \\
 &= q\alpha - p\alpha + 2p\alpha \\
 &= q\alpha + p\alpha = \alpha
 \end{aligned}$$

$\Rightarrow \alpha$ is called Allele-Substitution effect

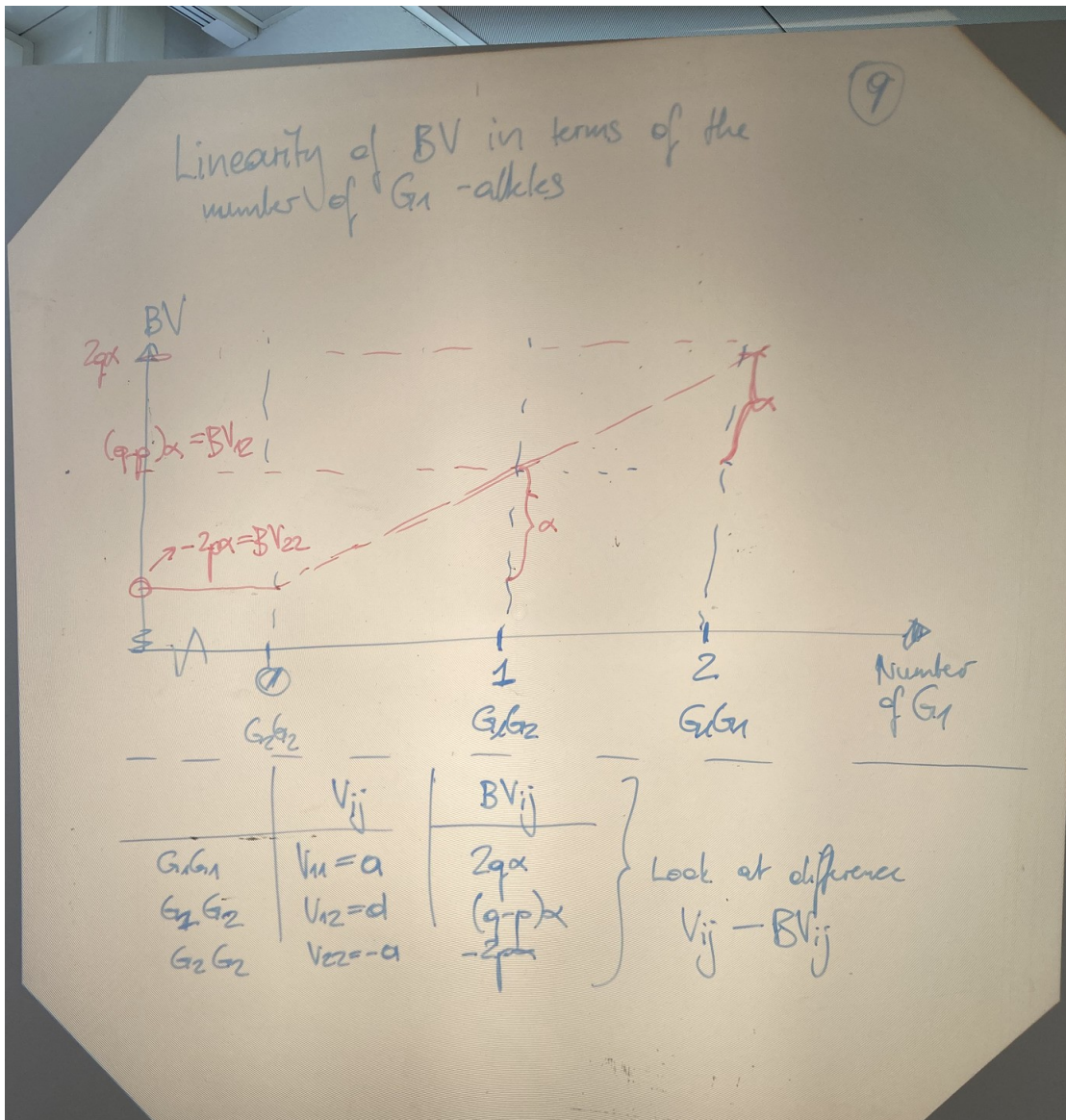
Animal S with $BV_{22} = -2p\alpha$

$BV_{12} = (q-p)\alpha$

PCRISPR

$\Delta BV = \alpha$

OHP Picture 9



OHP Picture 10

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Difference: $V_{ij} - BV_{ij}$

$\Delta_{ij} = \frac{V_{ij} - BV_{ij}}{G_1 G_1} : G_1 G_1$

$\Delta_{11} = V_{11} - BV_{11} = a - 2q\alpha$

$= a - 2q(a + (q-p)d)$

$= a - 2qa + 2q(p-q)d$

$= a - 2qa + 2q^2d + 2pqd$

$= a(1-2q) - 2q^2d + 2pqd$

$= \underbrace{[(p-q)a + 2pqd]}_{\mu} - \underbrace{2q^2d}_{D_{11}}$

$= \mu + D_{11} ; \text{ where } D_{11} = -2q^2d$

$G_1 G_2: \Delta_{12} = V_{12} - BV_{12} = d - (q-p)\alpha = \dots$

$= (p-q)a + 2pqd + 2pqd$

$= \mu + 2pqd$

$= \mu + D_{12}$

$G_2 G_2: \Delta_{22} = V_{22} - BV_{22} = \dots = \mu + D_{22}; D_{22} = -2p^2d$

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Summary

	V_{ij}	BV_{ij}	D_{ij}
$G_1 G_1$	$V_{11} = a$	$2pq\alpha$	$-2q^2d$
$G_1 G_2$	$V_{12} = d$	$(q-p)\alpha$	$2pqd$
$G_2 G_2$	$V_{22} = -a$	$-2pq\alpha$	$-2p^2d$

→ Dominance deviation

Decomposition of Genotypic Values

$$\Delta_{11} = V_{11} - BV_{11} = \mu + D_{11}$$

$$\Delta_{12} = V_{12} - BV_{12} = \mu + D_{12}$$

$$\Delta_{22} = V_{22} - BV_{22} = \mu + D_{22}$$

$$\Delta_{ij} = V_{ij} - BV_{ij} = \mu + D_{ij}$$

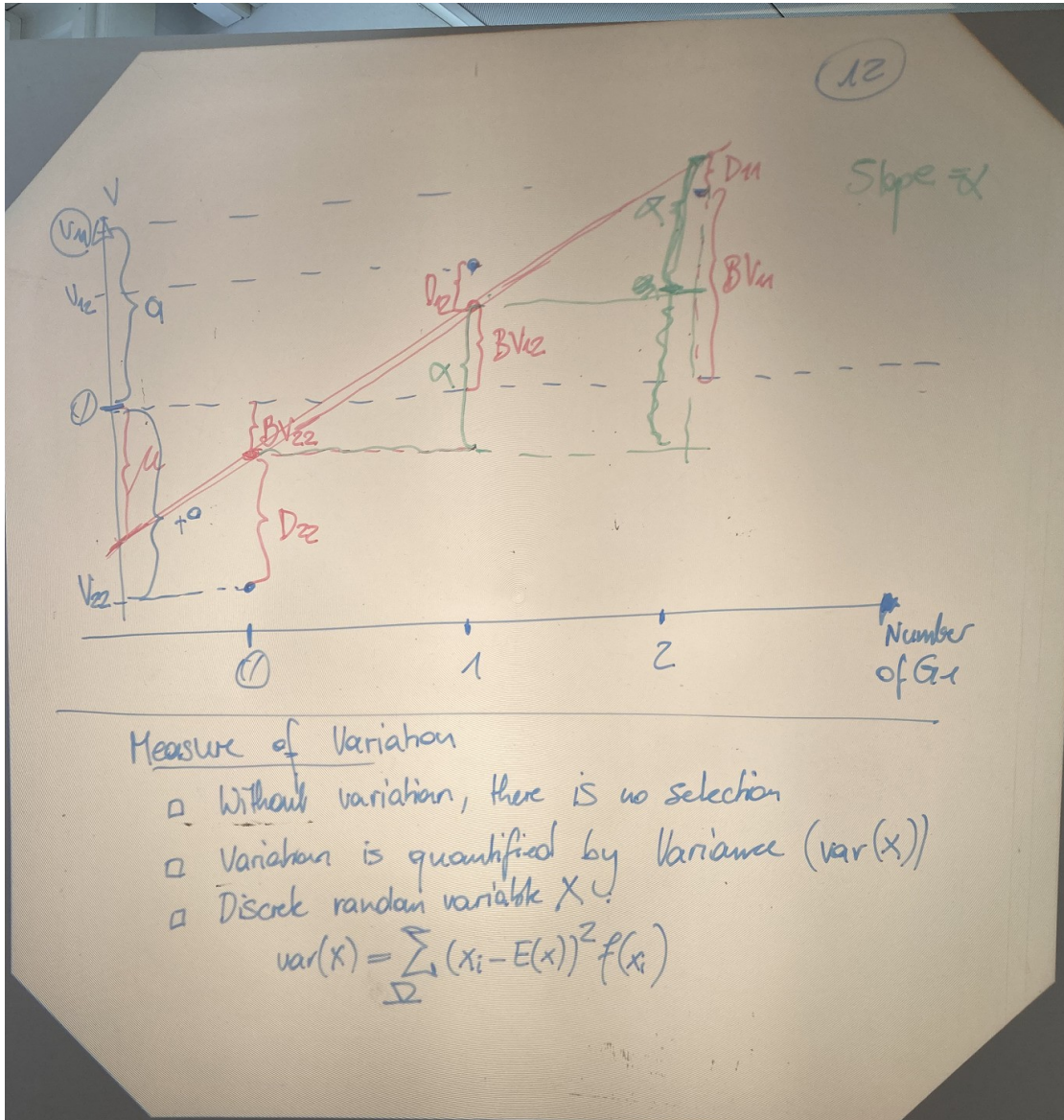
$$V_{ij} = \mu + BV_{ij} + D_{ij}$$

Intercept Predictor Residual

Interpreted as Regression Model

Response

OHP Picture 12



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Variance of genotypic values V

$$\begin{aligned} \sigma_g^2 \text{var}(V_{ij}) &= (V_{11} - \mu)^2 f(G_1 G_1) \\ &+ (V_{12} - \mu)^2 f(G_1 G_2) \\ &+ (V_{22} - \mu)^2 f(G_2 G_2) \\ &= (V_{11} - \mu)^2 p^2 + (V_{12} - \mu)^2 2pq + (V_{22} - \mu)^2 q^2 \\ &= (a - [(pq)a + 2pgd])^2 p^2 \\ &+ (d - [(pq)a + 2pgd])^2 2pq \\ &+ (-a - [(pq)a + 2pgd])^2 q^2 \end{aligned}$$

Use decomposition:

$$\begin{aligned} \sigma_g^2 \text{var}(V_{ij}) &= (BV_{11} + D_{11})^2 p^2 + (BV_{12} + D_{12})^2 2pq + (BV_{22} + D_{22})^2 q^2 \\ &= \dots \\ &= 2pgd^2 + (2pgd)^2 \\ &= \underbrace{\sigma_A^2}_{\text{additive genetic variance}} + \underbrace{\sigma_D^2}_{\text{dominance variance}} \end{aligned}$$

$$\begin{aligned} \text{var}(BV_{ij}) &= (BV_{11} - E(BV))^2 p^2 + (BV_{12} - E(BV))^2 2pq + (BV_{22} - E(BV))^2 q^2 \\ &= BV_{11}^2 p^2 + BV_{12}^2 2pq + BV_{22}^2 q^2 = \sigma_A^2 \end{aligned}$$

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Decomposition:

$$V_{ij} = \mu + BV_{ij} + D_{ij} \rightarrow \text{Regression with BV as predictor on D as residual}$$

• Var on both sides

$$V_G^2 = \text{var}(V_{ij}) = \text{var}(\mu + BV_{ij} + D_{ij})$$

$$= \text{var}(\mu) + \text{var}(BV_{ij}) + \text{var}(D_{ij})$$

$$+ 2\text{cov}(\mu, BV_{ij}) + 2\text{cov}(\mu, D_{ij})$$

$$+ 2\text{cov}(BV_{ij}, D_{ij}) = 0$$

$$= \text{var}(BV_{ij}) + \text{var}(D_{ij})$$

$$= \sigma_A^2 + \sigma_D^2$$

$Y = \mu + \beta x + \epsilon$
 $\text{cov}(x, \epsilon) = 0$