# Henderson's Rule

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2024-11-22

# Aim

- Use simple example pedigrees to establish Henderson's rules to construct $A^{-1}$
- Trivial example pedigree: 3 founder animals

```
##   sire  dam
## 1 <NA> <NA>
## 2 <NA> <NA>
## 3 <NA> <NA>
```

# Computation of $A^{-1}$

$$A^{-1} = (L^{-1})^T \cdot D^{-1} \cdot L^{-1}$$

with

- $L^{-1} = I - P$
- $P$ is the transformation matrix from simple decomposition: $u = P \cdot u + m$
- diagonal elements of $D$ obtained from `pedigreemm::Dmat()`

# Example Pedigree

- Diagonal elements of $D$

```
## 1 2 3
## 1 1 1
```

- Matrix $P$ is the zero matrix $\rightarrow$ matrix $L = I$
- Matrix $A^{-1} = I$
- First part of the rule: For animals without parents add element $(D^{-1})_{ii}$ to each diagonal element $(A^{-1})_{ii}$

# Extend Pedigree

- Add animal 1 as parent for animal 3

```
##   sire  dam
## 1 <NA> <NA>
## 2 <NA> <NA>
## 3    1 <NA>
```

# Matrix Setup

- Diagonals of matrix $D^{-1}$ yield the following matrix

```
##        [,1] [,2]     [,3]
## [1,]     1    0 0.000000
## [2,]     0    1 0.000000
## [3,]     0    0 1.333333
```

- Matrix $L^{-1} = I - P$

```
##        [,1] [,2] [,3]
## [1,]   1.0    0    0
## [2,]   0.0    1    0
## [3,]  -0.5    0    1
```

# Computation of $A^{-1}$

$D^{-1}$

| | [,1] | [,2] | [,3] |
|---|---|---|---|
| [1,] | 1 | 0 | 0.000000 |
| [2,] | 0 | 1 | 0.000000 |
| [3,] | 0 | 0 | 1.333333 |

$L^{-1}$

| | [,1] | [,2] | [,3] |
|---|---|---|---|
| [1,] | 1.0 | 0 | 0 |
| [2,] | 0.0 | 1 | 0 |
| [3,] | -0.5 | 0 | 1 |

| | [,1] | [,2] | [,3] |
|---|---|---|---|
| [1,] | 1 | 0 | -0.5 |
| [2,] | 0 | 1 | 0.0 |
| [3,] | 0 | 0 | 1.0 |

$(L^{-1})^T$

| | [,1] | [,2] | [,3] |
|---|---|---|---|
| [1,] | 1 | 0 | -0.6666667 |
| [2,] | 0 | 1 | 0.0000000 |
| [3,] | 0 | 0 | 1.3333333 |

| | [,1] | [,2] | [,3] |
|---|---|---|---|
| [1,] | 1.3333333 | 0 | -0.6666667 |
| [2,] | 0.0000000 | 1 | 0.0000000 |
| [3,] | -0.6666667 | 0 | 1.3333333 |

$A^{-1}$

# Extension of Rule

- So far: Add $(D^{-1})_{ii}$ to $(A^{-1})_{ii}$
- Check for animals 1, 2 and 3
- New:
  - Subtract $(D^{-1})_{ii}/2$ from $(A^{-1})_{is}$ and $(A^{-1})_{si}$
  - Add $(D^{-1})_{ii}/4$ to $(A^{-1})_{ss}$

# Second Parent

- Add animal 2 as second parent of animal 3

```
##   sire  dam
## 1 <NA> <NA>
## 2 <NA> <NA>
## 3    1    2
```

- Do the same setup of matrices and describe the extensions of the rule as exercise